

A SIMPLE DIGITAL MODEL OF THE DIODE-BASED RING-MODULATOR

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ABSTRACT

The analog diode-based ring modulator has a distinctive sound quality compared to standard digital ring modulation, due to the non-linear behaviour of the diodes. It would be desirable to be able to recreate this sound in a digital context, for musical uses. However, the topology of the standard circuit for a diode-based ring modulator can make the process of modelling complex and potentially computationally heavy. In this work, we examine the behaviour of the standard diode ring modulator circuit, and propose a number of simplifications that maintain the important behaviour but are simpler to analyse. From these simplified circuits, we derive a simple and efficient digital model of the diode-based ring modulator based on a small network of static non-linearities. We propose a model for the non-linearities, along with parameterisations that allow the sound and behaviour to be modified dynamically for musical uses.

1. INTRODUCTION

Ring-modulation (RM) (also known as four-quadrant modulation) is a technique similar to amplitude modulation (AM) and frequency modulation (FM) and was developed, like AM and FM, for radio transmission applications. Ideal RM is a special case of AM (i.e. the multiplication of a carrier and modulator signal), where both the carrier signal and the modulator signal are centred around 0V. When this condition is fulfilled, the carrier and modulator signals are completely cancelled and the output of the system consists of only the sum and difference frequencies of these inputs. This effect is musically useful, because it allows harmonic sounds to be transformed into clangorous inharmonic sounds whilst still retaining some of their original character and articulation.

Ring modulation was first used as musical effect by German avant-garde composers [1], notably Karlheinz Stockhausen. Ring modulators were also used by the BBC's Radiophonic Workshop in the 1950s and 1960s, notably to produce the distinctive voice of the 'Daleks' in the television show 'Doctor Who' [2]. Established as a normal part of the early electronic music studio, ring modulators were naturally included in early analog modular synthesizers. Don Buchla included a ring-modulator as one of the first modules of his System 100 synthesizer, built for Morton Subotnick [3]. Robert Moog's initial complement of modules did not include a ring-modulator, but one was later added via a collaboration with Harald Bode. Thus, the ring-modulator came to be an element available on a significant proportion of analog synthesizers, both modular and non-modular.

The name 'ring-modulation' refers to the way in which this technique was often implemented in its early days, using a configuration that employed a 'ring' of diodes [4] [5]. A schematic of this

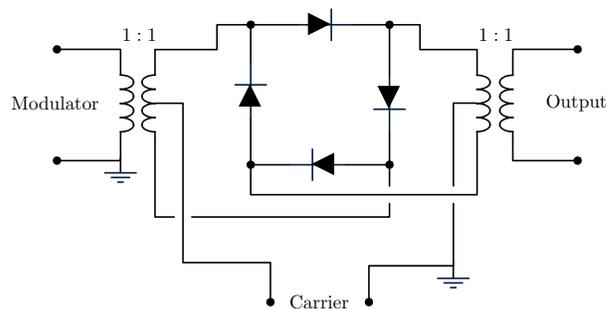


Figure 1: Schematic showing the layout of a traditional ring-modulator circuit.

type of ring-modulation circuit can be seen in Figure 1. The diodes used in the circuit have a strong effect on its final behaviour. The use of silicon diodes results in a hard clipping or 'chopping' effect and hence extremely bright extra harmonics, whilst germanium diodes produce a softer non-linearity and a 'warmer' sound [1]. Later ring modulators instead employed a structure based on VCAs, producing an output with far less added harmonics [5].

The basic idea of ring-modulation is trivial to implement digitally, as it involves only a simple multiplication of two signals. However, such a digital ring modulator lacks the additional non-linear behaviour (beyond the multiplication itself) and extra generated harmonics of a real analog ring modulator, and hence a large part of its characteristic sound. It is therefore desirable to produce a digital model that can replicate this behaviour to some extent. Previous work has derived ordinary differential equations governing the ring-modulator circuit, and solved them numerically using the Forward Euler method [6]. However, this method requires significant over-sampling (a factor of $\times 128$ is suggested), and is hence not ideal for real-time usage.

In Section 2 of this paper, we propose a simplified circuit which behaves analogously to the traditional diode ring-modulator. In Section 3 we discuss how this circuit can be modelled digitally using a network of static non-linearities. In Section 3.1 we discuss how the shape of these non-linearities can be derived, and in Section 3.2 we discuss the results produced by this algorithm. Finally, in Section 4, we conclude.

2. A SIMPLIFIED RING MODULATOR CIRCUIT

Previous work has suggested that majority of the distortion characteristics of the diode ring-modulator can be explained by assuming that only two of the diodes in the ring conduct at any one time, the

particular pair selected by the polarity of the carrier voltage [7] [8]. It is then postulated that distortion of the signal is produced by the non-linear characteristics of the diodes, and by the finite time it takes to switch between pairs [7] [8]. However, the presence of transformers in the circuit described in Figure 1 complicates the behaviour of the circuit in a number of ways, primarily by coupling voltage between nodes of the diode ring and causing, in some circumstances, three diodes to conduct simultaneously. This behaviour can be confirmed easily by measurement or by simulation in a package such as SPICE or Qucs [9].

We propose a simplified circuit which does not contain a transformer. By examining the circuit in Figure 1, we can see that if the modulator waveform is denoted V_{in} and the carrier waveform denoted V_c , the voltage at the two input nodes of the diode ring should be $V_c + V_{in}/2$ and $V_c - V_{in}/2$. We therefore replace the input transformer and V_{in} and V_c voltage sources with two voltage sources at these nodes providing the combined voltages. We then replace the output transformer with two resistors connected to ground. Figure 2 shows a schematic of this circuit. By examining this circuit, it should be clear that the diode pair D_1 and D_3 conduct when the signal V_c is positive, and the diode pair D_2 and D_4 conduct when it is negative. The signal V_{in} is present at both opposite nodes of the diode-ring, in opposing polarities, but is of smaller amplitude and assumed to not bias the diodes. The output is taken across the two resistors R_{out} (i.e. voltage $v_2 - v_4$), both of which have a high resistance value. The resistors R_{in} have a low resistance.

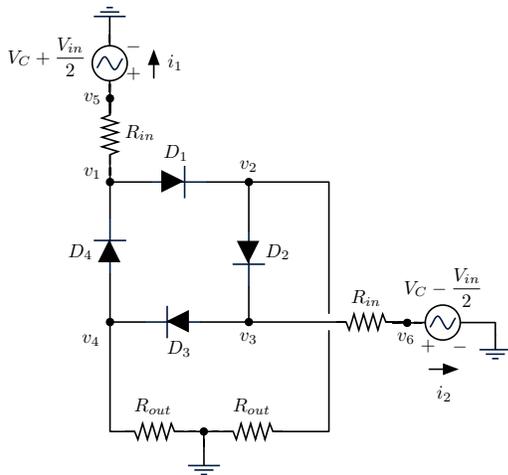


Figure 2: Schematic showing the layout of the simplified, transformer-less ring-modulator circuit.

We then make the assumption that current only flows through two diodes simultaneously, controlled by the polarity of V_c [7] [8]. We then assume that the closed diodes act as an open circuit and hence produce two substitute circuits representing the state of the circuit when V_c is positive, and when it is negative. Figure 3 shows a schematic of these circuits.

3. MODELLING THE SIMPLIFIED CIRCUIT

By examining the simplified circuit, it is easy to see what kind of digital signal processing structure could be used to replicate the

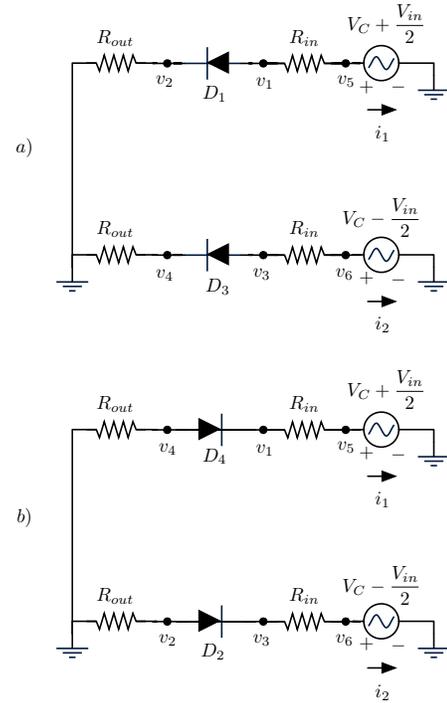


Figure 3: Schematic showing the layout of the simplified, transformer-less ring-modulator circuit when a) V_c is positive and b) V_c is negative.

circuit. There are four parallel signal paths, two which process the combination $V_c + V_{in}/2$ and two which process $V_c - V_{in}/2$. Each of these signal paths consists of a non-linearity representing the voltage-to-voltage transfer function of the resistor-diode-resistor chain, plus (as necessary) inversion to represent the direction of the diode and whether the branch of the circuit ends on v_2 or v_4 . A block diagram showing this structure is given in Figure 4. The next task is then to derive the form of the non-linearity used on each of the parallel paths of the structure.

3.1. Diode Non-Linearity Model

One possible method of modelling the diode non-linearity present in each of the branches of the simplified circuit is to use standard nodal analysis to solve for the voltage at v_2 or v_4 in one of the branches. We do this by modelling the diode using Shockley's ideal diode equation, which is given by:

$$i_D = I_S \left(e^{\frac{v_D}{nV_T}} - 1 \right) \quad (1)$$

where i_D is the current through the diode, I_S is the reverse bias saturation current, v_D is the voltage across the diode, V_T is the thermal voltage and n is the ideality factor.

The exponential functions introduced by the use of Shockley's diode equation produce a relationship between the input and output voltage of the diode that is only solvable analytically by application of the Lambert W-function. Instead we expand out the exponentials using Taylor series, and then solve the resulting implicit relationship. The input-output voltage relationship of the diode when modelled in this way is given in Figure 5. The parameter values used to calculate this curve are given in Table 1, and are

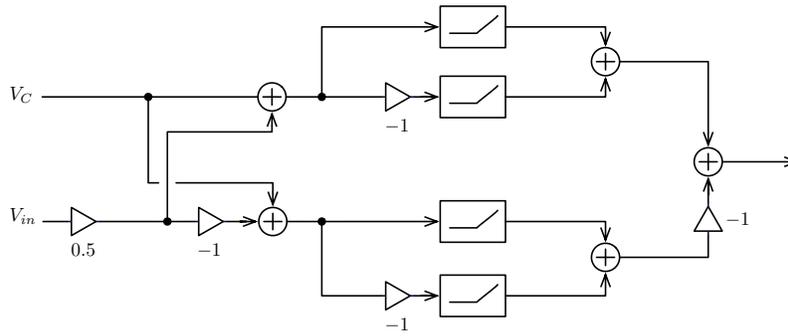


Figure 4: Block diagram showing the signal processing structure used to model the simplified ring modulator circuit.

chosen to be consistent with an average germanium diode such as the 1N34.

Table 1: Values used for the fixed parameters of the model when calculating the shaping function.

Section	Element	Value
Diodes	n	2.19
	V_T	26×10^{-3}
	I_S	10^{-12}
Fixed	R_{in}	80
Resistors	R_{out}	10^6

This method of calculating the non-linearity is only suitable for producing a look-up table, as it involves a large amount of computation. More desirable would be a non-linear function that requires few operations to calculate and which lends itself to variation by the user in order to adjust the sound of the effect. Looking at the form of the curve, it is clear that it is separated into three main sections. At low voltage, the output is approximately zero. At high voltages, the relationship is approximately affine. These sections are connected by a smooth curve. It is therefore possible to produce a very similar curve using a piecewise function, consisting of a zero section, a polynomial, and an affine section. This function is given by:

$$f(v) = \begin{cases} 0 & v \leq v_b \\ h \frac{(v-v_b)^2}{2v_L-2v_b} & v_b < v \leq v_L \\ hv - hv_L + h \frac{(v_L-v_b)^2}{2v_L-2v_b} & v > v_L \end{cases} \quad (2)$$

where v is the input voltage, v_b is a parameter specifying the equivalent of the diode forward bias voltage, v_L is a parameter giving the voltage beyond which the function is linear, and h is a parameter specifying the slope of the linear section. The shape of this function, with parameters adjusted to minimise maximum error within the region of interest compared to the modelled curve, is given in Figure 5.

3.2. Results

Figure 7 shows the output of the algorithm when driven by a 1V 500 Hz sinusoidal V_{in} and a 1V 1500 Hz sinusoidal V_C . Figure 6 shows the output of a real diode-based ring modulator, built to a design consistent with the schematic given in Figure 1 [10]. It is

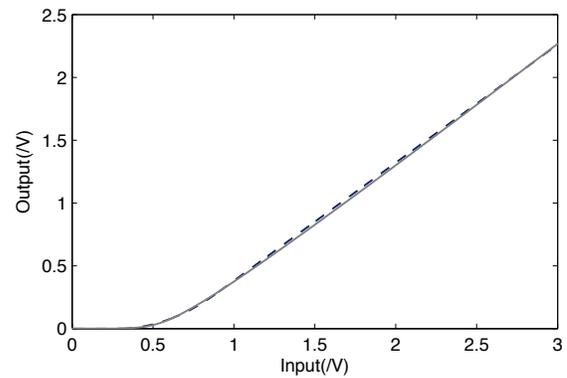


Figure 5: Graph showing the input-output relationship of the diode wave-shaper. The version derived from circuit analysis is given in solid grey, whilst the piecewise approximation is given as a dashed black line.

clear that whilst not identical, the output of the digital model is reasonably similar to the measured signal. There is a discrepancy in the peak voltage between the real and modelled ring-modulators. This is probably caused by the lack of buffering on the outputs of the real ring-modulator (as it is a passive circuit), which leads to the device having an output impedance that causes some signal loss when connected to a standard audio pre-amplifier.

Figure 8 shows the frequency spectrum of the output of model when presented with 50 Hz V_{in} and 1500 Hz V_C . As expected, visible are the sum and difference frequencies at 1550 Hz and 1450 Hz, along with further integer multiples of the modulator frequency above and below the carrier frequency. Also visible are odd harmonics of the modulated spectrum.

Informal listening tests show that the output of the algorithm is sonically satisfying when applied to synthetic material, with much of the character of a vintage ring-modulator present. When applied to more natural sound sources, for example the human voice, the result of the algorithm is harsher and brasher than that of simple digital multiplication, as would be expected. Variation of the parameters of the diode wave-shaping function, as given in Equation 2, allow a variety of distortion characters to be generated - from soft germanium-diode style saturation to the harsher distortion associated with silicon diodes. Sound examples of the algorithm applied to both synthetic and natural sources are available at the website associated with this work [11]. Evaluation of the exact accuracy of the algorithm compared to both the analog ring-modulator

and to direct numerical modelling of the circuit [6] would require further investigation. However, the author would like to emphasise that aim of the algorithm is to produce a computationally cheap vintage ring-modulator style effect, rather than to replicate the behaviour of the analog circuit exactly.

The algorithm is efficient, and could comfortably be run on modern computers in real-time with a small cost in computational load. The structure itself requires 10 operations per sample, along with 4 calls to the diode-shaping function. The diode-shaping function requires 14 operations each time it is called. In systems where it is cheap to calculate the absolute value of a sample (for example by discarding the sign of a floating point number), both the $V_C + V_{in}/2$ and $V_C - V_{in}/2$ paths can be reduced to using a single nonlinearity, by proceeding the nonlinearity with a call to an $abs()$ or equivalent function.

As with any non-linear algorithm, some oversampling is recommended to avoid aliasing of the generated higher harmonics. Examination of the harmonics generated by the system reveals that they fall-off at a rate of around 20db per octave. An oversampling factor of around $\times 32$ would therefore be necessary to suppress aliasing if wide-band signals were used for both the modulator and carrier. In practical applications, the carrier is often sinusoidal, and the modulator may be a voice or instrumental signal for which the majority of the energy is in the band below 4-5kHz. In these applications, an oversampling factor of $\times 16$ or $\times 8$ is sufficient to avoid audible aliasing. The aliasing characteristics of the algorithm could be improved by re-designing the diode non-linearity function as a pure polynomial function instead of a piecewise function, and this will be an interesting topic for further work.

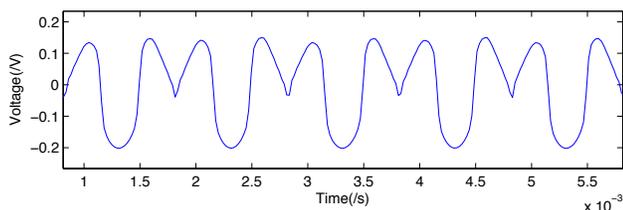


Figure 6: Output voltage produced when the real ring-modulator circuit is driven by a 500 Hz modulator and a 1500 Hz carrier at 1V

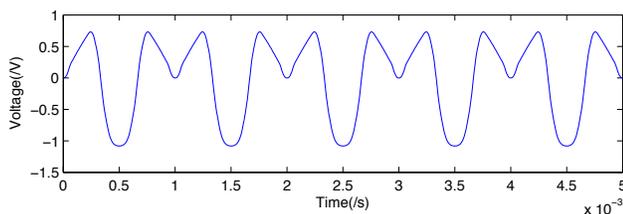


Figure 7: Output voltage produced when the model is used to process a 500 Hz modulator and a 1500 Hz carrier at 1V.

4. CONCLUSIONS

In this work we examined the operation of the diode-based ring-modulator, and proposed a simplified analogous circuit that does not contain transformers. This analogous circuit produces similar behaviour to the traditional diode ring-modulator, but its structure

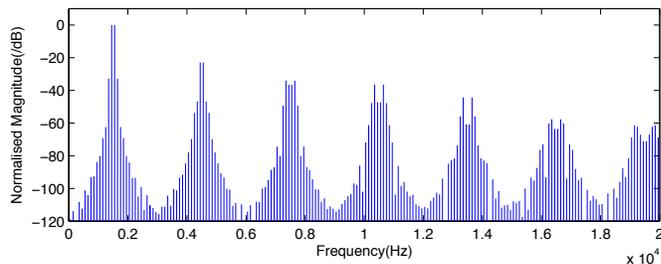


Figure 8: Spectrum of the output of the model when used to process a 50 Hz modulator and a 1500 Hz carrier at 1V.

makes it simpler to model. We then showed how this structure could be modelled digitally using a simple network of static nonlinearities. The result is an efficient parametric ring-modulator effect, which is suitable for real-time use in a computer music environment.

5. ACKNOWLEDGMENTS

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