THE PLUCKSYNTH TOUCH STRING

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ABSTRACT

In this paper the problem of the synthesis of plucked strings by means of physically inspired models is reconsidered in the context of the player's interaction with the virtual instrument. While solutions for the synthesis of guitar tones have been proposed, which are excellent from the acoustic point of view, the problem of the control of the physical parameters directly by the player has not received sufficient attention. In this paper we revive a simple model previously presented by Cuzzucoli and Lombardo for the player's touch. We show that the model is affected by an inconsistency that can be removed by introducing the finger/pick perturbation in a balanced form on the digital waveguide. The results, together with a more comprehensive model of the guitar have been implemented in a VST plugin, which is the starting point for further research.

1. INTRODUCTION

The plucked string is often regarded as the simplest musical acoustic system, which is amenable to an easy-to-implement physically inspired model for sound synthesis. However, this assertion is only true in a very crude approximation, which, besides disregarding the non-linearities, does not take into account the interaction of the player's finger, or of the pick, with the string. Modeling and controlling the player's touch is indeed a complex problem that to date has only found partial solution. The practical system proposed in [1] totally refrains from modeling this interaction by introducing, instead, two databases of string excitation and damping signals that are applied as input to the string Karplus-Strong like filter via a pluck-shaping filter in cascade with a comb filter modeling the plucking point. Although the quality of the acoustic result justifies the method, in applications where the interaction of the player with the string plays an essential role in the sound control interface, the proposed system is not usable.

In [2] Cuzzucoli and Lombardo introduce an interesting and simple model for the interaction of the finger/pick with the string. The proposed model is linear and derives the shape of the excitation from the coupling of two dynamical systems, the string and the finger/pick. The models proposed in [3] for the force exerted by the player, which include friction between the nail/pick and the string, help shaping the control functions of the model.

Unfortunately, the published version of [2] contains a few obvious typos and, in our point of view, also a model inconsistency which makes the use of special tricks necessary for retriggering the played string. In this paper we present a correction of the Cuzzucoli-Lombardo model and point out some of its extensions. In a recent paper [4] a modal approach to the synthesis of guitar pluck has been presented. In the paper no model for the interaction of the player with the string is reported. The model illustrated in this paper in the context of digital waveguide synthesis is also useful in the context of modal synthesis.

The player-string interaction model, completed by an acoustic model of the guitar body, has been implemented in a VST plugin, which includes a specific user interface for controlling the model parameters, also illustrated in the paper. The plugin has been developed for research purposes and is freely available at http://staffwww.itn.liu.se/∼giaev/soundexamples.html.

2. THE CUZZUCOLI-LOMBARDO MODEL

In this section we review the model of interaction of the finger/pick with the string presented in [2]. The finger/pick is represented by a damped spring-mass system, defined by a mass M, a spring stiffness K and a damping coefficient R. When plucking the string, the finger/pick is in contact (solidal) to a string segment, positioned on the interval $[x_1, x_2]$, of length $\Delta = x_2 - x_1$ and linear mass density μ. The remaining portions on the left and on the right of the plucking segment behave like two strings, each attached to one termination (nut or bridge) and both coupled to the plucked segment. Within the strings, the transversal waves propagate as in a waveguide and reflect at the boundaries. As a result of plucking, an incremental deformation of the string is produced over the plucking segment, which propagates along the strings.

Let $y = y(x, t)$ denote the deformation of the string with respect to the equilibrium position. At the plucking segment, the following forces are available:

- the external force $F_0(t)$ exerted by the player
- the restoring force $-Ky$ of the finger/pick (spring model)
- the damping term $-R\frac{\partial y}{\partial t}$ of the finger/pick
- the resultant of the transversal component of the tensile force of the string $F(t)$ acting at the extreme points of the plucking segment.

These forces balance the force of inertia $(M + \mu\Delta)\,\frac{\partial^2 y}{\partial t^2}$ of the mass system composed by the finger/pick and the string segment in contact with it. Therefore, the equilibrium equation can be written as follows:

$$(M + \mu\Delta)\,\frac{\partial^2 y}{\partial t^2} + R\frac{\partial y}{\partial t} + Ky - F(t) = F_0(t) \qquad (1)$$

In the linear approximation of the string, valid for small deformations, the force $F(t)$ is given by

$$F(t) = K_0 \left(\left.\frac{\partial y}{\partial x}\right|_{x=x_2} - \left.\frac{\partial y}{\partial x}\right|_{x=x_1} \right), \qquad (2)$$

where K_0 is the tension of the string, which is assumed to be constant. When the interaction of the finger/pick with the string is lumped to a single point, a discontinuity of the first spatial derivative of the deformation of the string is introduced at the plucking point similar to that caused by a concentrated force applied to the point (see, for example, [5] p. 19).

The main control function of the model is the player's force $F_0(t)$. However, in order to model the movement and detachment of the finger/pick from the string, time varying mass M, damping coefficient R and stiffness K functions must be assigned, which fast decay to 0 immediately after the pluck phase is over. In order to be consistent, the model should revert to that of a freely oscillating string when these parameters vanish.

A discrete-time model of the pluck interaction can be derived from (1) and (2), by space-time sampling the solution $y(x,t)$ and replacing the derivatives with finite differences. Away from the plucking segment the classical wave equation for the flexible string holds, whose solution can be written in terms of propagating waves. Given a temporal sampling interval T and propagation velocity $c = \sqrt{K_0/\mu}$ along the string, the spatial sampling interval is, as is common practice, chosen as $X = cT$, which simplifies the form of the discrete traveling wave solution. The length of the string L is assumed to be an integer multiple N of the spatial sampling interval: $L = NX$. Tuning the discrete model of the string also requires the use of additional fractional delays, which we will disregard in these preliminary considerations. Central differences are introduced to approximate the time derivatives:

$$\left.\frac{\partial y}{\partial t}\right|_{x=nX,\, t=mT} \leftrightarrow \frac{y(n,m+1)-y(n,m-1)}{2T}$$
$$\left.\frac{\partial^2 y}{\partial t^2}\right|_{x=nX,\, t=mT} \leftrightarrow \frac{y(n,m+1)-2y(n,m)+y(n,m-1)}{T^2} \tag{3}$$

Assuming, for simplicity, that $\Delta = X$ and that the plucking segment coincides with the interval $\left[\left(n_p-\frac{1}{2}\right)X, \left(n_p+\frac{1}{2}\right)X\right]$, where $n_p < N$ is a positive integer, a discrete counterpart of (2) is

$$F(m) = \frac{K_0}{X}\left[y(n_p+1,m) - 2y(n_p,m) + y(n_p-1,m)\right], \tag{4}$$

where the spatial derivatives are replaced by central differences, centered on half-integer multiples of X.

With (3) and (4), the discretization of (1) yields the following finite difference equation:

$$\begin{aligned} c_1 y(n_p,m+1) - c_0 y(n_p,m) + c_{-1} y(n_p,m-1) \\ = y(n_p+1,m) + y(n_p-1,m) + \tfrac{X}{K_0}F_0(m), \end{aligned} \tag{5}$$

where

$$\begin{aligned} c_1 &= \left(1 + \tfrac{M}{\mu X} + \rho\right) \\ c_0 &= \left(\tfrac{2M}{\mu X} - \kappa\right) \\ c_{-1} &= \left(1 + \tfrac{M}{\mu X} - \rho\right) \end{aligned} \tag{6}$$

and we defined the adimensional parameters

$$\begin{aligned} \rho &= \frac{R}{2\sqrt{\mu K_0}} \\ \kappa &= \frac{KX}{K_0}. \end{aligned} \tag{7}$$

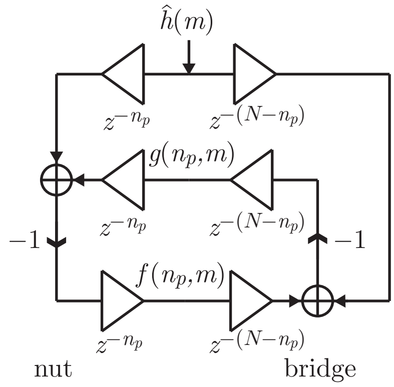

Figure 1: *A three delay line structure to compute plucked string oscillations.*

The structure shown in Figure 1 and consisting of three delay lines has been proposed in [2] in order to compute the solution of (5) together with the connected wave equation for the rest of the string. The frequency dependent losses of the string are temporarily left out of our discussion. In order to fix our ideas, we assume that the nut is placed at the leftmost extreme of the waveguide and the bridge at the rightmost extreme, where the nut stands from the actual guitar nut or the fret bar for non open-string sounds. Two of the three delay lines implement the right and left propagation of waves along the string as in a conventional 1D waveguide. The third delay line consists of two delay lines, one oriented toward the left and one toward the right of the plucking segment. Both these sub-lines host the propagation of the perturbation caused by the pluck excitation, respectively, from the left of the plucking segment to the nut and from the right of the plucking segment to the bridge. At the terminations, the perturbation wave is mixed with the inpinging wave from the conventional waveguide, $g(0,m)$ at the nut and $f(N,m)$ at the bridge, and reflected together. The deformation of the string at any point along the string is obtained by adding the content of the three lines at the given point. The perturbation $\hat{h}(m)$ at the plucking point, which will propagate along the upper line, is computed from the state of the three-line waveguide at the plucking point and at adjacent spatial locations. However, it easy to see that the three delay line scheme is equivalent to the two delay line structure shown in Figure 2, where, with reference to Figure 1, we let

$$\begin{aligned} \hat{y}^+(n,m) &= f(n,m) + \hat{h}(m-n+n_p), \text{ for } n > n_p \\ \hat{y}^+(n,m) &= f(n,m), \text{ for } n \leqslant n_p \\ \hat{y}^-(n,m) &= g(n,m) + \hat{h}(m+n-n_p), \text{ for } n < n_p \\ \hat{y}^-(n,m) &= g(n,m), \text{ for } n \geqslant n_p. \end{aligned} \tag{8}$$

An update equation for the perturbation $\hat{h}(m)$ can be obtained from (5) by relating the terms at location different from n_p to those at location n_p. From the diagram in Figure 2 we have:

$$\begin{aligned} y(n_p+1,m) &= \hat{y}^-(n_p,m+1) + \hat{y}^+(n_p,m-1) \\ &\quad + \hat{h}(m-1) \\ y(n_p-1,m) &= \hat{y}^-(n_p,m-1) + \hat{y}^+(n_p,m+1) \\ &\quad + \hat{h}(m-1). \end{aligned} \tag{9}$$

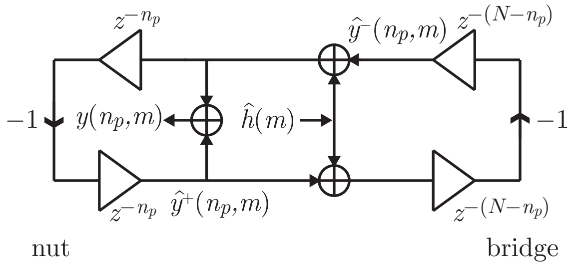

Figure 2: *Equivalent two delay line structure to compute plucked string oscillations.*

Thus,

$$y(n_p+1,m)+y(n_p-1,m)=y(n_p,m+1)-\hat{h}(m+1)$$
$$+y(n_p,m-1)+\hat{h}(m-1) \quad (10)$$

and substitution in (5) yields:

$$\hat{h}(m+1)=\hat{h}(m-1)+(1-c_1)y(n_p,m+1)+c_0y(n_p,m)$$
$$+(1-c_{-1})y(n_p,m-1)+\tfrac{X}{K_0}F_0(m). \quad (11)$$

For the sake of computation, the term in $y(n_p, m+1)$ can, in turn, be rewritten in terms of $\hat{h}(m+1)$ and $\hat{y}^+(n_p,m+1)$ and $\hat{y}^-(n_p,m+1)$, which yields a causal recurrence for $\hat{h}(m)$. However, from (11) we immediately note the following inconsistency: when the contact of the finger/pick with the string is removed, i.e., when M, K, R and F_0 all go to zero then the coefficients $1-c_{\pm 1}$ and c_0 also go to zero. In this case we are left with the recurrence $\hat{h}(m+1)=\hat{h}(m-1)$, which admits the constant or alternating solutions. Therefore, as the finger/pick is removed, the perturbation term $\hat{h}(m)$ does not necessarily vanish. Indeed, in our experimentation with the model we determined that as the finger/pick is removed before the string oscillation is fading away, the sequence $\hat{h}(m)$ is likely to get stuck or oscillate between two non-zero values, as required in order to provide the accurate solution of (5). This poses control problems when replucking the string: all perturbations caused by previous plucks are still active when a new note is played on the same string.

Although tricks can be devised to fade away the perturbation when the string needs to be replucked [6], discontinuities of the waveform are generated when the string is plucked at different locations. These are due to the fact that the wave variables $\hat{y}^+$ and $\hat{y}^-$ tend to be discontinuous around the plucking segment, although, due to cancellation, their sum is continuous. When the perturbation is artificially faded, these discontinuities present themselves as transients on the waveguide. The effect of discontinuities can be mitigated by the use of velocity waves [7], which do not depend on constant deformations. However the extra control function to drive the perturbation to zero is quite arbitrary and unnecessary. In fact, in the following section a method for the solution which does not suffer from these problems is presented. While the derivation can be performed using impedance models, as described in [8], we will take a finite difference approach in order to compare the model with that proposed in [2] and illustrated in this section.

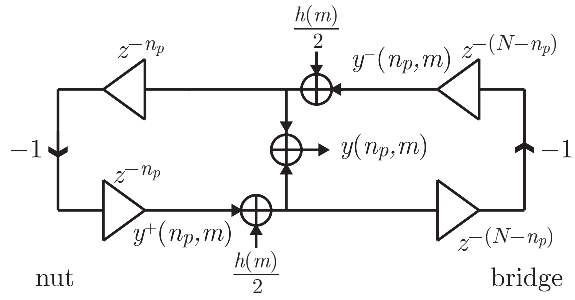

Figure 3: *Diagram of the balanced waveguide for the plucking interaction.*

3. IMPROVING THE PLAYER'S TOUCH MODEL

A simple and effective way to fix the inconsistency of the Cuzzucoli-Lomabardo player's touch model discussed in the previous section is to introduce the plucking perturbation in balanced form on the digital waveguide. This is achieved by means of the diagram in Figure 3, where the perturbation $h(m)$ is split in half and injected in the digital waveguide in a symmetric way on the two delay lines, on each side of the adder providing $y(n_p, m)$. Consequently, the propagation equations (9) are replaced by

$$y(n_p+1,m)=y^-(n_p,m+1)+y^+(n_p,m-1)+\tfrac{h(m-1)}{2}$$
$$y(n_p-1,m)=y^-(n_p,m-1)+\tfrac{h(m-1)}{2}+y^+(n_p,m+1) \quad (12)$$

and the analogon of (10) for the balanced waveguide is

$$\begin{aligned} &y(n_p+1,m)+y(n_p-1,m)\\ &=y(n_p,m+1)-h(m+1)+y(n_p,m-1). \end{aligned} \quad (13)$$

An important consequence is that now

$$\begin{aligned} h(m+1)&=(1-c_1)y(n_p,m+1)+c_0y(n_p,m)\\ &+(1-c_{-1})y(n_p,m-1)+\tfrac{X}{K_0}F_0(m), \end{aligned} \quad (14)$$

which implies that $h(m)$ vanishes as soon as the finger/pick contact with the string is removed, i.e., when M, K, R and F_0 all go to zero.

Notice that although both wave variables y^+ and y^- and perturbation $h(m)$ differ from the wave variables $\hat{y}^+$ and $\hat{y}^-$ and perturbation $\hat{h}(m)$ of Figure 2, they still yield the same solution $y(n,m)$ of the plucked string problem. The reason why this wave representation is more convenient is that no special measure has to be taken in order to control and fade out the perturbation when re-triggering a note played on the same string.

A causal update equation for the perturbation $h(m)$ can be derived from (14) by expressing all the terms in $y(n_p, r)$, for $r = m-1, ..., m+1$, in terms of the wave variables and perturbation at instant r. Passing to the z-transform, we obtain:

$$H(z)=\tfrac{B(z)}{A(z)}\left(Y^+(z)+Y^-(z)\right)+\tfrac{Xz^{-1}}{K_0A(z)}\tilde{F}_0(z), \quad (15)$$

where $Y^+(z)$, $Y^-(z)$ and $\tilde{F}_0(z)$ respectively are the z-transforms of $y^+(n_p,m)$, $y^-(n_p,m)$ and $F_0(m)$. For the polynomials $A(z)$ and $B(z)$ we have the following expression:

$$\begin{aligned} A(z)&=c_1-c_0z^{-1}-(1-c_{-1})z^{-2}\\ B(z)&=(1-c_1)+c_0z^{-1}+(1-c_{-1})z^{-2}. \end{aligned} \quad (16)$$

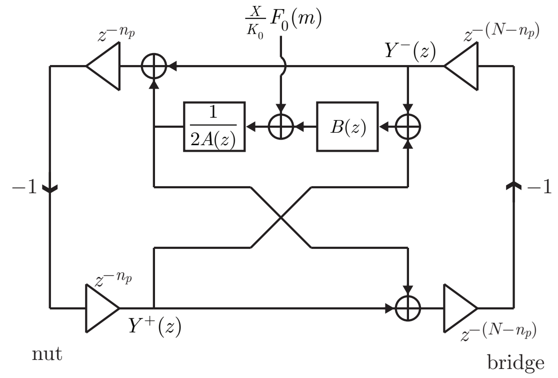

Figure 4: *Diagram of the balanced waveguide including the complete scattering junction for the plucking interaction.*

From (15) and the diagram of Figure 3 one obtains the diagram of Figure 4, where the pluck interaction is represented as a scattering junction [8] with force input, and the extra unnecessary delay in the force path has been removed. Notice that the numerator $B(z)$ of the scattering wave transfer function tends to 0 when the finger/pick is removed, while the numerator $A(z)$ tends to 1. As a result, after plucking, the propagation along the line reverts to that of an unperturbed single string.

3.1. Velocity Wave Variables

The obtained structure can be converted for use with velocity wave variables in place of displacement waves. In this case, the wave variables can be first integrated with a leaky integrator filter to obtain displacement, then the scattering junction processing is performed and the result is differentiated in order to obtain the velocity increment due to plucking. Since integration and differentiation along the path approximately cancel each other, the procedure is, in principle, equivalent to using for the velocity waves the same scattering junction as in Figure 4, provided that the force input is differentiated. Alternately, the force input can be described in terms of its time derivative, i.e., yank. However, notice that the scaling factor $X/2K_0$ in the force path converts force into displacement [8], where the factor $1/2$ is borrowed from the denominator $2A(z)$. Since $X/2K_0 = T/2Z_0$, where $Z_0 = \sqrt{\mu K_0}$ is the impedance of the string, converting force into velocity, then the input can be written in terms of the velocity v injected in the string by striking it with force F_0. The factor $1/2$ takes into account the fact that two portions of the string (left and right) are "seen" at the excitation point, acting as impedances connected in series. Hence $XF_0(m)/2K_0 = v(m)T$. Differencing this term yields $2(v(m) - v(m-1))$ to be presented as input at the scattering junction, in place of $XF_0(m)/K_0$, when velocity wave variables are chosen for the waveguide.

3.2. Transfer Function and Stability

The overall structure in Figure 4 including string waveguide, excitation scattering junction and boundary scatterers is a linear system, which can be described by a transfer function. However, the parameters of the scattering junction, such as finger mass, damping and stiffness, depend on time and are non-zero only during the excitation time interval. Therefore, the transfer function is time varying. After removal of the string-finger contact, the structure behaves like a waveguide modeling stationary waves in the string. Including losses, modeled as a gain factor $g \leq 1$ for each delay element of the waveguide, from the diagram of Figure 4 we obtain

$$Q(z) = \frac{Xz^{-(N-n_p)}(1 - g^{2n_p}z^{-2n_p})}{K_0\left[2A(z)(1 - g^{2N}z^{-2N}) + B(z)P(z)\right]}, \tag{17}$$

with

$$P(z) = g^{2(N-n_p)}z^{-2(N-n_p)} - 2g^{2N}z^{-2N} + g^{2n_p}z^{-2n_p}, \tag{18}$$

where we retained as output the forward propagating wave signal $y^+(N,m)$ at the bridge. The transfer function is still comb-like, as that of a Karplus-Strong string model to which it reverts when $A(z) = 1$ and $B(z) = 0$, but the presence of the polynomials $A(z)$, $B(z)$ and $P(z)$ has the effect of spreading some energy of the excitation to frequencies lying in between the teeth of the comb, as shown in Figure 5. This is observed in guitars and other instruments, where, in the attack phase, energy in between the harmonics is present, which decays rapidly after the initial transient. At the onset, the waveform looks like a set of pulse trains superimposed to the forming oscillation. In the model this is mainly due to the force input filtered by $1/2A(z)$ in the excitation path of the waveguide (see Figure 4). The filter implements a passive damped mass-spring system. Although we did not obtain the discrete-time filter by applying the bilinear transform to the Laplace version of the differential equation (1) – which guarantees stability but introduces frequency warping – the filter can be shown to be stable for all ranges of the physical parameters.

In order to establish string oscillations, the system typically operates in under-damped regime. This is reflected by complex conjugate poles responsible for the bandpass frequency response shown in Figure 6. The discriminant of the polynomial $A(z)$ is:

$$D = -\frac{M(\kappa+1)}{\mu X} + \frac{k^2}{4} + \rho(\rho+1). \tag{19}$$

In the under-damped regime $D < 0$ and the magnitude square of the two complex conjugated poles $z_\pm$ is:

$$|z_\pm|^2 = \frac{M - \rho\,\mu\,X}{M + (1+\rho)\,\mu\,X}, \tag{20}$$

which is clearly smaller than 1, so that the filter is stable. However, for a small damping coefficient ρ and mass of the finger/pick M much larger than the mass μX of the plucked segment of the string, the poles can get close to the unit circle. As a result, in a finite arithmetic implementation, suitable scaling is necessary in order to prevent overflow throughout the valid range of values of the physical parameters.

The dependency of frequencies $f_\pm$ of the poles on the physical parameters is the following:

$$f_\pm = \pm\frac{1}{2\pi T}\tan^{-1}\frac{\sqrt{\frac{M(\kappa+1)}{\mu X} - \frac{k^2}{4} - \rho(\rho+1)}}{\frac{M}{\mu X} - \frac{k}{2}}. \tag{21}$$

When both damping and stiffness parameters are negligible with respect to the mass ratio, we have:

$$f_\pm \approx \pm\frac{1}{2\pi T}\sqrt{\frac{\mu X}{M}}, \tag{22}$$

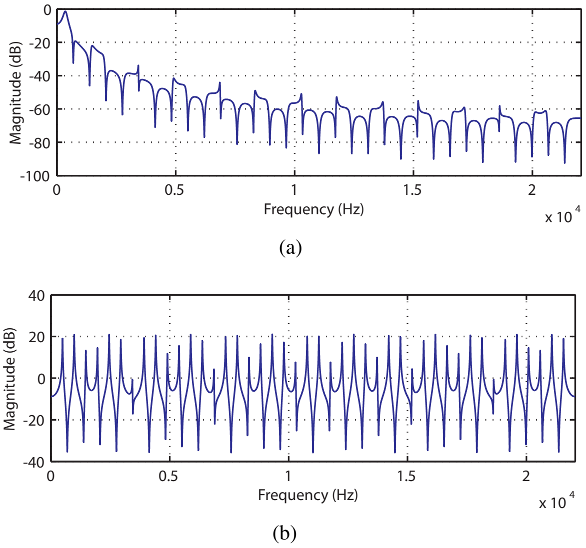

Figure 5: *Typical frequency response of string waveguide structure: (a) during excitation; (b) during free oscillation.*

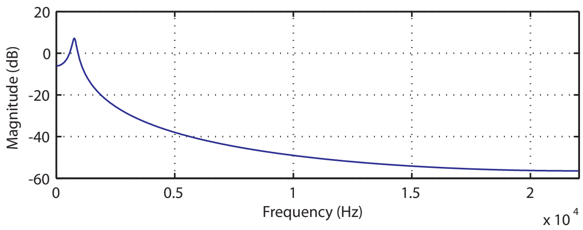

Figure 6: *Typical frequency response of the filter* $\frac{1}{2A(z)}$ *in the excitation path of the pluck model.*

which shows that the peak frequency of the bandpass filter $1/2A(z)$ decreases as the finger/pick mass grows.

The filter $B/2A(z)$ in the wave path of the pluck scattering junction in Figure 4 typically has a high-pass response, almost flat in the passband, as shown in Figure 7. The phase response of the filter is however such that the net transfer function $1+B(z)/2A(z)$ in the wave path is slightly low-pass, with a few dB attenuation in the stopband, as shown in Figure 8.

In realistic models of the string, the elementary losses g are frequency dependent. They are consolidated at the boundaries of the string and implemented as low-pass filters, one at each end of the waveguide (loop filters). In our scheme, the frequency response of the loop filters varies according to the position of the pluck point along the string. This effect is simulated by employing as loop filters two low-pass filters with variable cut-off frequency. The overall waveguide comb filter operates at the margins of stability in order to produce sustained oscillations. The loop filters (consolidated gains) determine the frequency dependent decay rate of the tone partials. The bridgeside loop filter includes an additional filter modeling the scattering of the waves at an elastic bridge.

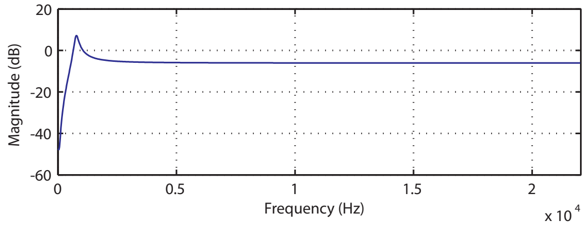

Figure 7: *Typical frequency response of the filter* $\frac{B(z)}{2A(z)}$ *in the wave path of the pluck model.*

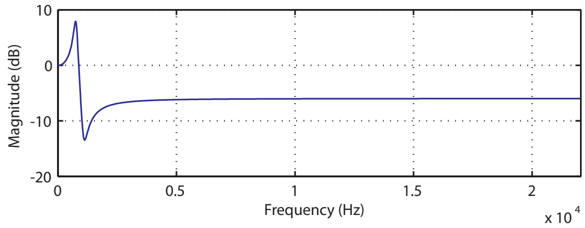

Figure 8: *Typical frequency response of the complete filter* $1 + \frac{B(z)}{2A(z)}$ *in the wave path of the pluck model.*

4. PLAYING AND CONTROLLING THE STRING INTERACTION MODEL

A major asset in enforcing a physical model for the finger/pick interaction with the string lies in the fact that simple functions, related to the force and contact dynamics, can be employed to feed the waveguide and obtain realistic sounds. The major benefit obtained from putting the Cuzzucoli-Lombardo model in consistent form is that re-plucking the string, especially at different positions, does not involve special care, otherwise necessary in order to drive the string wave variables to a rest state. In the improved model, when force, finger/pick mass, damping and stiffness control signals all go to zero, the string reverts to a stationary wave oscillating state without showing discontinuities of the wave variables along it.

Simple control functions, as the ones shown in Figure 9 can be employed to effectively excite the string, with good acoustical results. In the variation of the physical parameters we distinguish a pre-damping phase, in which the finger/pick comes in contact with the string and mostly damps residual oscillations, an excitation phase, where the player applies force to the string through the finger/pick (with possibly time varying contact characteristics) and a detach phase, where the contact of the finger/pick with the string is removed. In the pre-damping phase, increasing damping $R(t)$ is applied until a limit level is reached. This phase can last a few milliseconds and can be much shorter than the one shown in the figure. The main purpose of the pre-damping phase is to control the plucking style: a longer pre-damping phase features a "softer" playing style. After pre-damping, a gradually increasing force $F_0(t)$ is applied to the string and, simultaneously, both the finger/pick inertia, parametrized by the mass $M(t)$, and elastic recoil forces, characterized by the stiffness coefficient $K(t)$, are applied to the string. These quantities vary in an arbitrary fashion and much depend on the pluck style (apoyando, tirando, etc.) and medium (finger, nail or plectrum). Constant stiffness and mass

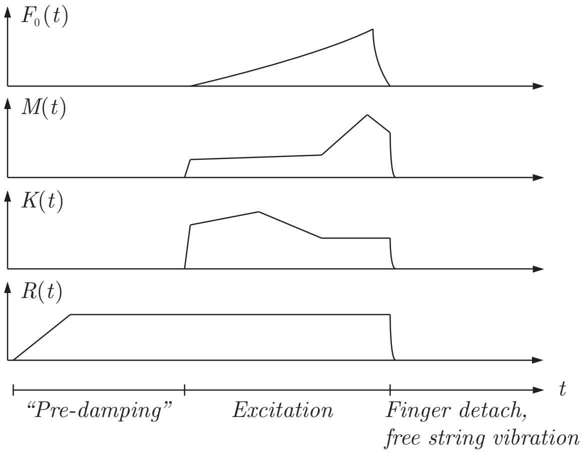

Figure 9: *Time-line of simple control functions for the physical parameters.*

are most likely not representative of a real pluck. For example, the stiffness can be considered to increase as a soft plectrum is deformed during contact, or vary when the player switches from driving the string with the finger to grab it with the nail, a situation when also the mass parameter can be assumed to change. A non-linear sping model can also be employed, similar to the felt hammer model in the synthesis of piano. Finally, in the detach phase, all external forces abruptly decay to zero, leaving the string free to vibrate.

The force exerted on a guitar string may be represented by a two-segment signal. During the initial attack phase, the force rises from zero to a final maximum and then rapidly goes back to zero as the string is released. Depending on how the player approaches the string, the force curve is here assumed to take on different shapes and time durations. Two types of curves are used for the construction of force signal segments: an "S-shaped" one and an exponential curve with variable decay. Denoting the attack and release time intervals, respectively, by t_a and t_r, and given the maximum force magnitude A_f, the "S-shaped" force segment curves are given by

$$F(t) = \begin{cases} A_f \, \frac{1}{2} \left[1 - cos\left(\pi \frac{t}{t_a}\right)\right] & \text{for } 0 \leq t \leq t_a \\ A_f \, \frac{1}{2} \left[1 + cos\left(\pi \frac{t-t_a}{t_r}\right)\right] & \text{for } t_a < t \leq t_a + t_r \\ 0 & \text{for } t > t_r + t_a \end{cases}, \tag{23}$$

while the exponential curves are given by

$$F(t) = \begin{cases} A_f \, \left(\frac{t}{t_a}\right)^{s_a} & \text{for } 0 \leq t \leq t_a \\ A_f \, \left(\frac{t-t_a}{t_r}\right)^{s_r} & \text{for } t_a < t \leq t_a + t_r \\ 0 & \text{for } t > t_r + t_a \end{cases} \tag{24}$$

The parameters s_a and s_r determine the curve slopes: if $s_a = s_r = 1$ a linear segment is obtained. Modifying these slope parameters together with t_a and t_r, a range of differently shaped force inputs can be generated. The segment shapes may also be combined, so a linear attack segment may for example precede an "S-shaped" release. Three possible variations are depicted in Figure 10.

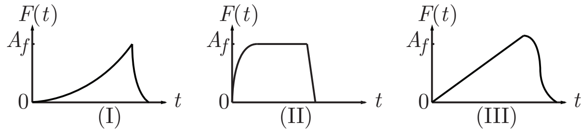

Figure 10: *(I) Exponential segments with s_a and $s_r < 1$; (II) Exponential attack and linear release with $s_a > 1$ and $s_r = 1$; (III) Linear attack segment with "S-shaped" release segment.*

If the release time is too long, the string will be unnaturally damped during the detach phase, as if the string were slowly released and never really given any excitation. As in [2], proper release times is taken to be around one half a period of the strings' fundamental frequency. The finger mass and stiffness coefficients are obtained by amplitude modulating their peak values, respectively M_a and K_a. The corresponding control signals are calculated as follow:

$$\begin{aligned} M(t) &= M_a \times M_{mod}(t) \\ K(t) &= K_a \times K_{mod}(t) \end{aligned} \tag{25}$$

where $M_{mod}(t)$ and $K_{mod}(t)$ are piecewise linear functions composed of 4 segments (similar to an ADSR envelope generator), followed by a final release segment with exponential decay. Methods involving different equations for generating time varying parameters are of course possible, our choice rested on the convenience of the user interface. In our scheme, the damping parameter ρ remains constant during the action of the force. Although it is possible to modulate this parameter as well, but this was not found to make a particularly significant perceptual difference to the produced sound.

The approach taken to tune the simulated string and make the guitar model playable was to add a third termination to the string simulation, placed in between the nut and bridge terminations. The termination structure and control setup mimics a guitar player's finger action on the fretboard. The structure used is similar to the scattering junction illustrated in Section 3, with the exclusion of the external force term. Statically, this force is perfectly balanced by the reaction of the fret once the finger is pressed against it (tapping the string is not modeled here). Muted playing or stopping the string at the fret can also be simulated by means of this junction [7]. Structurally, the finger on fret junction is equivalent to the one-filter dynamic scattering junction in [9], with the addition of impedances due to the damping and elastic terms. As positioning of the fret junction is only possible at a distance integer multiple of the spatial sampling X, an all-pass filter is added in cascade to the loop filters at the bridge in order to fine tune the tones by introducing a fractional delay.

5. THE PLUCKSYNTH USER INTERFACE

The PluckSynth guitar pluck synthesizer has been implemented within the SynthEdit environment [10], with C++ modules interfaced with the SynthEdit SDK. A graphical user interface (GUI),

Figure 11: *The control panel of the PluckSynth VST plugin.*

built around standard control blocks, allows for the real/time control of the meaningful parameters. Some of these parameters, such as damping, can also be controlled via MIDI input and assigned to gesture controllers, such as wheel or joystick. The main control panel of the PluckSynth GUI is shown in Figure 11.

The system implements the waveguide based string interaction model described in Section 3, complete of loop filters at the terminations, fine-tuning allpass filter and fret model. A linear superposition of two waveguides is employed for each string, one modeling vertical vibration and one representing perpendicular horizontal movement (polarization). We enforced the approximation that the planes of vibration are linearly coupled and that coupling occurs at the bridge position [11]. In order to extend the excitation model for use with a two polarizations string model, the string feedback is taken as a linear combination of the two waveguides. The computed excitation signal $h(m)$ is distributed to the waveguides through an angle of string approach parameter "Angle", in the range 0 to 1 and controlled by the corresponding knob, using $h_{vert}(m) = Angle \times h(m)$ and $h_{horiz}(m) = (1 - Angle) \times h(m)$. During the excitation, this causes an additional coupling point between the two polarizations to be present at the plucking position. The audible effect depends upon the settings for the sustain/bridge polarization coupling. Separate knobs available in the GUI allow us to control the sustain of each mode and the amount of coupling between vertical and horizontal planes. An additional "inharmonicity" knob controls the detuning of the vibrations in the two planes, allowing for the introduction of beating or chorus effects. Allpass interpolated delay lines are employed here to insert in the waveguides a MIDI controlled time varying delay equivalent to a maximum pitch bend of one semitone. Sympathetic coupling among different strings is not currently implemented in the system.

The user can choose between electric or acoustic guitar. In the former case two moving magnetic pick-ups can be placed at will between the bridge and the last fret on the fretboard. A simple pickup model is used. Magnetic guitar pickups are sensitive to the string velocity and behave much like resonant low pass filters. Hence, a pickup is simply an observation point in the velocity wave delay lines, whose output is filtered with a 12db/octave resonant low pass filter. This filter cutoff and merit factor determines the sound character of pickup. For the acoustic guitar, an empirical model of the body is provided, which includes a number of filters, each of which representing a resonant mode of the body. The initial values of the center frequencies, gains and roll-off factors of these filters were obtained by matching the measured frequency response of a specific acoustic guitar. In a secondary control panel of the user interface, these values can be altered at will or loaded from file. Moreover, according to their center frequencies, the body filters are grouped into three bands ("Low", "Mid" and "High"), which allow the user to control the overall characteristic by changing the gain factor of each group. A "brightness" control, using standard equalization techniques, is also included. Presets concerning the string type, from nylon to metal can also be selected in the panel.

The parameters of the finger/pick interaction with the string can be selected in the upper-right section of the GUI. There, two xy-pads control the force signal segments: the "relax" axis controls, respectively, the attack and release time, while the "style" axis controls the shape of the segments, respectively s_a and s_r in (24). If the cursor is positioned at a zero value on the style axis, the force excitation segment coincides with the half period cosine function of Equation (23). The finger mass and stiffness peak values and modulations in (25) can be controlled respectively by two knobs and two editable piecewise linear curves. Two sliders controlling the amount of modulation are available.

Pre-damping time and amount of damping, as discussed in Section 4, are controlled by two knobs in the GUI. In order to simulate the friction of the finger/pick sliding on the string, in our implementation we also added a noise term on the force excitation, whose amount can be controlled by the "scratch" knob. Different surfaces are represented by a set of sampled noise types, selectable using the "pick surface" presets. A more accurate friction model is currently at study that should allow us to describe finger and string surface characteristics on a physical basis.

In the PluckSynth implementation, the final detach phase (see Figure 9) is governed by an additional parameter α. The detach starts at the end of the forced excitation, where the physical parameters F_0, M, K and R may still have a non-zero value. In this phase the value of these parameters is recursively multiplied by a parameter $0<\alpha<1$ at each time step. This has been found to alter the string shape around the excitation position as the excitation model is detached from the string. Setting α to 0 yields a "pointy" string shape, increasing α towards 1 causes the string shape to be smoother, resulting in a softening of the perceived plucked string sound. In the absence of a spatially distributed excitation, the α parameter is used to conveniently model the shape of the finger/plectrum and is selected by means of the "shape" knob in the GUI.

The "attack" knob of the GUI controls the amount of longitudinal modes. These modes, leading to the presence of phantom (inharmonic and high pitched) partials, are computed as described in [7], inspired by methods found in [12, 13].

A fret change on the same string can be generalized as raising the finger, relocating it and pushing it down again, a sequence taking place during a time span of a few milliseconds up to a second, depending on the players style. In addition, a player may change a string from open state to fret terminated state and vice versa. To mimic such transitions, the parameters of the finger-on-fret model are varied over time using curves similar to the S-shaped functions given in (23). The corresponding controls are located in the GUI just above the fretboard.

The plucksynth features two playing modes. In addition to the "Solo" mode, where the instrument is played similar to an ordinary keyboard instrument, a "Chord" mode was also implemented. In the leftmost bottom corner of the user interface, a set of 12 chords can be pre-defined. A special keyboard layout becomes active in which the lower octave (player's left hand) is used to select and tune the strings to form a chord from the pre-defined set, while the upper octave (player's right hand) excites the strings [7]. With the "pb" toggle buttons located in the leftmost part of the fretboard display, the user can choose which strings will be affected by the pitch bend MIDI signal, making it possible to pitch bend only particular strings inside a chord.

6. CONCLUSION

A refined model of the player's touch on guitar strings has been presented in this paper. The model improves playability of the model in [2], which suffers from control inconsistencies due to a peculiar choice of the wave variables. The model, together with other state of the art or approximated for real-time computation models for the various components of electric or acoustic guitars has been implemented in an experimental VST plugin. It can be evaluated that the acoustical results are quite realistic. Further studies and implementations are envisaged in order to better model friction on the string and to efficiently incorporate stiffness and non-linearities of the string model, which would improve realism especially for the lower pitched tones.

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