

## SELF-AUTHENTICATION OF AUDIO SIGNALS BY CHIRP CODING

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### ABSTRACT

This paper discusses a new approach to ‘watermarking’ digital signals using linear frequency modulated or ‘chirp’ coding. The principles underlying this approach are based on the use of a matched filter to provide a reconstruction of a chirped code that is uniquely robust in the case of signals with very low signal-to-noise ratios.

Chirp coding for authenticating data is generic in the sense that it can be used for a range of data types and applications (the authentication of speech and audio signals, for example). The theoretical and computational aspects of the matched filter and the properties of a chirp are revisited to provide the essential background to the method. Signal code generating schemes are then addressed and details of the coding and decoding techniques considered. Finally, the paper briefly describes an example application which is available on-line for readers who are interested in using the approach for audio data authentication working with either WAV or MP3 files.

### 1. INTRODUCTION

Digital watermarking has been researched for many years in order to achieve methods which provide both anti-counterfeiting and authentication facilities [1]. One of equations that underpins this technology is based on the model a the signal given by (e.g. [2], [3] and [4])

$$s = \hat{P}f + n \quad (1)$$

where  $f$  is the information content for the signal,  $\hat{P}$  is a linear operator,  $n$  is noise and  $s$  is the output signal. This equation is usually taken to describe a stationary process which includes the characterisation of  $n$  (i.e. the probability density function of  $n$  is assumed to be invariant of time).

In the field of cryptography, the operation  $\hat{P}f$  is referred to as the processes of ‘diffusion’ and the process of adding noise (i.e.  $\hat{P}f + n$ ) is referred to as the process of ‘confusion’. The principal ‘art’ is to develop methods in which the processes of diffusion and confusion are maximized; one important criterion being that the output  $s$  should be dominated by the noise  $n$  which in turn should be characterized by maximum Entropy (i.e. a uniform statistical distribution) [6].

Instead of  $n$  being taken to be noise, suppose that  $n$  is a known signal and that  $\|n\| \gg \|\hat{P}f\|$ . In this case it may be possible to embed or ‘hide’ the information contained in  $f$  in the signal  $n$  without significantly perturbing it. The process of hiding secret information in signals or images is known as Steganography [5] and being able to recover  $f$  from  $s$  in equation (1) can provide a

way of authenticating the signal  $n$ . If, in addition, it is possible to determine that a copy of  $s$  has been made leading to some form of data degradation and/or corruption that can be conveyed through an appropriate analysis of  $f$ , then a scheme can be developed that provides a check on: (i) the authenticity of the data  $n$ ; (ii) its fidelity [7], [8]. In this case, signal  $f$  is an example of a watermark.

Formally, the recovery of  $f$  from  $s$  is based on the inverse process

$$f = \hat{P}^{-1}(s - n)$$

where  $\hat{P}^{-1}$  is the inverse operator. Clearly, this requires the signal  $n$  to be known *a priori* and that the inverse process  $\hat{P}^{-1}$  is well defined and computationally stable. Since the host signal  $n$  must be known in order to recover the watermark  $f$ , this approach leads to a private watermarking scheme in which the field  $n$  represents a key. In addition, the operator  $\hat{P}$  (and its inverse  $\hat{P}^{-1}$ ) can be key dependent. The value of this operator key dependency relies on the nature and properties of the operator that is used and whether it is compounded in an algorithm that is required to be in the public domain, for example.

Another approach is to consider the case in which the signal  $n$  is unknown and to consider the problem of extracting the watermark  $f$  in the absence of knowledge of this signal. In this case, the reconstruction is based on the result

$$f = \hat{P}^{-1}s + m$$

where

$$m = -\hat{P}^{-1}n.$$

If a process  $\hat{P}$  is available in which  $\|\hat{P}^{-1}s\| \gg \|m\|$ , then an approximate reconstruction of  $f$  may be obtained in which  $m$  is determined by the original signal-to-noise ratio of the data  $s$  and hence, the level of covertness of the information  $\hat{P}f$  - diffused watermark. In this case, it may be possible to post-process the reconstruction and recover a relatively high-fidelity version of the watermark, i.e.

$$f \sim \hat{P}^{-1}s.$$

This approach (if available) does not rely on a private key (assuming  $\hat{P}$  is not key dependent). The ability to recover the watermark only requires knowledge of the operator  $\hat{P}$  (and its inverse) and post-processing options as required. The problem is to find an operator that is able to diffuse and recover the watermark  $f$  effectively in the presence of the signal  $n$  when  $\|\hat{P}f\| \ll \|n\|$ , i.e. with very low signal-to-noise ratios. Ideally, we require an operator  $\hat{P}$  with properties such that  $\hat{P}^{-1}n \rightarrow 0$ .

In this paper, we consider the case where the operator  $\hat{P}$  is based on a chirp function, specifically, a linear Frequency Modulated (FM) chirp of the (complex) type  $\exp(i\alpha t^2)$  where  $\alpha$  is the

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chirp parameter and  $t$  is the independent variable<sup>1</sup>. This function is then convolved with  $f$ . The inverse process is undertaken by correlating with the (complex) conjugate of the chirp  $\exp(-i\alpha t^2)$ . This provides a reconstruction for  $f$  that is accurate and robust. Further, we consider a watermark based on a coding scheme in which the signal  $n$  is the input. The watermark  $f$  is therefore  $n$ -dependent. This allows an authentication scheme to be developed in which the watermark is generated from the signal in which it is to be 'hidden'. Authentication of the watermarked data is then based on comparing the code generated from  $s \sim n$  and that reconstructed from  $s = \hat{P}f + n$  when  $\|\hat{P}f\| \ll \|n\|$ . This is an example of a self-generated coding scheme which avoids the use, distribution and application of reference data. In this paper, the coding scheme is based on the application of Daubechies wavelets.

There are numerous applications of this technique in areas such as telecommunications and speech recognition where authentication is often mandatory. For example, as demonstrated in this paper, the method can be applied to audio data with no detectable differences in the audio quality of the data.

## 2. THE MATCHED FILTER AND LINEAR FM 'CHIRP' FUNCTIONS

The Matched Filter (e.g. [9], [10] and [11]) is one of the most common filters used for pattern recognition. It is based on correlating a signal/image with a matching template of the feature that is assumed to be present in the signal/image [4]. If the feature does indeed exist, then the output of the filter (the correlation signal/surface) produces a local maximum or spike where the feature occurs. This process can be applied generally, but when the template and feature are based on chirp functions, the result has some special and important properties which provide an output that is uniquely robust in the case when the signal-to-noise ratio is very low. It is this property that forms the basis for a variety of active imaging systems such as those used in Real and Synthetic Aperture Radar (e.g. [12], [13] and [14]), active sonar and some forms of seismic prospecting, for example. Interestingly, some mammals (including dolphins, whales and bats) use frequency modulation for communication and (target) detection. The reason for this is the unique properties that chirps provide in terms of the quality of extracting information from signals with very low signal-to-noise ratios and the simplicity of the process that is required to do this (i.e. correlation). The invention and use of chirps for man made information and communications recovery dates back to the early 1960s (the application of FM to radar, for example); 'mother nature' appears to have 'discovered' the idea some time ago.

### 2.1. The Matched Filter

We start by considering the basic linear stationary (convolution) model for a signal  $s$  as a function of time  $t$ , namely

$$s(t) = p(t) \otimes f(t) + n(t)$$

where  $p$  is the Impulse Response Function (IRF),  $f$  is the object function (the information content of some input signal),  $n$  is the noise (which is typically taken to have stationary statistics) and  $\otimes$  is the convolution operation, i.e.

$$p(t) \otimes f(t) = \int p(t - \tau)f(\tau)d\tau.$$

<sup>1</sup>In practice this is undertaken using the real or imaginary part of the complex chirp function.

A fundamental inverse (deconvolution) problem is to find an estimate  $\hat{f}$  of  $f$  given  $s$ . The Matched Filter is based on assuming a linear convolution model for this estimate of the form

$$\hat{f}(t) = q(t) \otimes s(t).$$

Clearly, the problem is to find the filter  $q$ . The Matched Filter is based on finding  $q$  subject to the condition that

$$r = \frac{|\int Q(\omega)P(\omega)d\omega|^2}{\int |N(\omega)|^2 |Q(\omega)|^2 d\omega} \quad (2)$$

is a maximum where  $Q$ ,  $P$  and  $N$  are the Fourier transforms of  $q$ ,  $p$  and  $n$  respectively and where we defined the Fourier transform pair as

$$F(\omega) = \int f(t) \exp(-i\omega t)dt,$$

$$f(t) = \frac{1}{2\pi} \int F(\omega) \exp(i\omega t)d\omega$$

in which the limits of the integrals are taken to be in  $(-\infty, \infty)$  and  $\omega$  is the (angular) frequency. Note that the ratio defining  $r$  is a 'measure' of the signal-to-noise ratio. In this sense, the matched filter maximizes the Signal-to-Noise Ratio (SNR) of the output.

Given equation (2), the matched filter is essentially a 'by-product' of the 'Schwarz inequality', i.e. the result

$$\left| \int Q(\omega)P(\omega)d\omega \right|^2 \leq \int |Q(\omega)|^2 d\omega \int |P(\omega)|^2 d\omega.$$

We write

$$Q(\omega)P(\omega) = |N(\omega)| |Q(\omega)| \times \frac{P(\omega)}{|N(\omega)|}$$

so that the above inequality becomes

$$\begin{aligned} \left| \int Q(\omega)P(\omega)d\omega \right|^2 &= \left| \int |N(\omega)| |Q(\omega)| \frac{P(\omega)}{|N(\omega)|} d\omega \right|^2 \\ &\leq \int |N(\omega)|^2 |Q(\omega)|^2 d\omega \int \frac{|P(\omega)|^2}{|N(\omega)|^2} d\omega. \end{aligned}$$

From this result, using the definition of  $r$  given in equation (2), we see that

$$r \leq \int \frac{|P(\omega)|^2}{|N(\omega)|^2} d\omega.$$

Now, if  $r$  is to be a maximum, then we require that

$$r = \int \frac{|P(\omega)|^2}{|N(\omega)|^2} d\omega$$

or

$$\begin{aligned} \left| \int |N(\omega)| |Q(\omega)| \frac{P(\omega)}{|N(\omega)|} d\omega \right|^2 \\ = \int |N(\omega)|^2 |Q(\omega)|^2 d\omega \int \frac{|P(\omega)|^2}{|N(\omega)|^2} d\omega. \end{aligned}$$

But this is only true if

$$|N(\omega)| |Q(\omega)| = \frac{P^*(\omega)}{|N(\omega)|}.$$

Hence,  $r$  is a maximum when

$$Q(\omega) = \frac{P^*(\omega)}{|N(\omega)|^2}.$$

Noise is usually characterised by: (i) the Probability Density Function (PDF) or the Characteristic Function (i.e. the Fourier transform of the PDF); (ii) the Power Spectral Density Function (PSDF). To apply the Matched Filter, the function  $|N(\omega)|^2$  (i.e. the power spectrum of the noise), in addition to  $P(\omega)$ , is required to be known *a priori*. In some practical systems, this is possible if the Impulse Response Function is zero so that the output of the system is ‘noise driven’. In general however, it is often necessary to develop a suitable model for the PSDF. Such models may include uniform, Gaussian, Poisson or random fractal noise, for example, which may be suitable in many cases [3]. However, if we consider the case when the PSDF is uniform or ‘white’ and of unit amplitude then we can write

$$|N(\omega)|^2 = 1 \forall \omega$$

so that the Matched Filter reduces to the simple result

$$Q(\omega) = P^*(\omega).$$

The required solution is therefore given by

$$\hat{f}(t) = \frac{1}{2\pi} \int P^*(\omega) S(\omega) \exp(i\omega t) d\omega.$$

Using the ‘correlation theorem’ we can write

$$\hat{f}(t) = p(t) \odot s(t) \equiv \int p(\tau + t) s(\tau) d\tau.$$

Hence, the matched filter is based on correlating the signal  $s$  with the instrument function  $p$ .

## 2.2. Deconvolution of Linear Frequency Modulated Chirps

The matched filter is frequently used in systems that utilize linear Frequency Modulated (FM) signals. Signals of this type are known as ‘chirped signals’. A linear FM signal which is taken to be of compact support ( $t \in [-T/2, T/2]$ ) is given (in complex form) by

$$p(t) = \exp(i\alpha t^2), \quad |t| \leq \frac{T}{2}$$

where  $\alpha$  is a constant (this defines the ‘chirp rate’) and  $T$  is the length of the signal. The phase of this signal is given by  $\alpha t^2$  (i.e. it has a quadratic phase factor) and its instantaneous frequency is therefore given by

$$\frac{d}{dt}(\alpha t^2) = 2\alpha t$$

which varies linearly with time  $t$ . Hence, the frequency modulations are linear which is why the signal is referred to as a ‘linear’ FM pulse.

For the purpose of clarity, let us first consider the case when the additive noise term is neglected and consider a signal given by

$$s(t) = \exp(i\alpha t^2) \otimes f(t), \quad |t| \leq \frac{T}{2}.$$

If we now apply a (white noise) matched filter, then we have

$$\hat{f}(t) = \exp(-i\alpha t^2) \odot \exp(i\alpha t^2) \otimes f(t), \quad |t| \leq \frac{T}{2}.$$

The correlation integral can now be evaluated thus

$$\exp(-i\alpha t^2) \odot \exp(i\alpha t^2) = \int_{-T/2}^{T/2} \exp[-i\alpha(\tau+t)^2] \exp(i\alpha\tau^2) d\tau$$

$$= \exp(-i\alpha t^2) \int_{-T/2}^{T/2} \exp(-2i\alpha\tau t) d\tau$$

Evaluating the integral over  $\tau$ , we have

$$\exp(-i\alpha t^2) \odot \exp(i\alpha t^2) = T \exp(-i\alpha t^2) \text{sinc}(\alpha T t)$$

and hence

$$\hat{f}(t) = T \exp(-i\alpha t^2) \text{sinc}(\alpha T t) \otimes f(t).$$

A further useful simplification can now be made to the result for  $\hat{f}$  which allows the exponential term to be ignored. In particular, if we consider  $T \gg 1$  then

$$\cos(\alpha t^2) \text{sinc}(\alpha T t) \simeq \text{sinc}(\alpha T t)$$

and

$$\sin(\alpha t^2) \text{sinc}(\alpha T t) \simeq 0$$

so that

$$\hat{f}(t) \simeq T \text{sinc}(\alpha T t) \otimes f(t)$$

This simplification, under a condition that is usually practically applicable, allows the result for  $\hat{f}$  to be easily analysed in Fourier space. Using the convolution theorem we can write (ignoring scaling by  $\pi/\alpha$ )

$$\hat{F}(\omega) = \begin{cases} F(\omega), & |\omega| \leq \alpha T; \\ 0, & |\omega| > \alpha T. \end{cases}$$

which describes  $\hat{f}$  as being a band-limited version of  $f$  (assuming the  $f$  is not band-limited) where the bandwidth is determined by  $\alpha T$ .

In the presence of additive noise, the result is

$$\hat{f}(t) \simeq T \text{sinc}(\alpha T t) \otimes f(t) + \exp(-i\alpha t^2) \odot n(t).$$

The correlation function produced by the correlation of  $\exp(-i\alpha t^2)$  with  $n(t)$  will in general be relatively low in amplitude since  $n(t)$  will not normally have features that match those of a (complex) chirp. Thus, it is reasonable to assume that

$$\|T \text{sinc}(\alpha T t) \otimes f(t)\| \gg \| \exp(-i\alpha t^2) \odot n(t) \|$$

and that in practice,  $\hat{f}$  is a band-limited reconstruction of  $f$  with high SNR. Thus, the process of using chirp signals with matched filtering for the purpose of reconstruction in the presence of additive noise provides a relatively simple and computationally reliable method of ‘diffusing’ and reconstructing information encoded in the function  $f$ . This is the underlying principle behind the method of watermarking described in this paper.

## 3. CHIRP CODING, DECODING AND WATERMARKING

We now consider the an approach to watermarking signals using chirp functions. The basic model for the watermarked signal (which is real) is

$$s(t) = \text{chirp}(t) \otimes f(t) + n(t)$$

where

$$\text{chirp}(t) = \sin(\alpha t^2)$$

We consider the field  $n(t)$  to be some pre-defined signal to which a watermark is to be 'added' to generate  $s(t)$ . In principle, any watermark described by the function  $f(t)$  can be used. On the other hand, for the purpose of authentication we require two criterion: (i)  $f(t)$  should represent a code which can be reconstructed accurately and robustly; (ii) the watermark code should be sensitive (and ideally ultra-sensitive) to any degradation in the field  $n(t)$  due to lossy compression and/or copying. To satisfy condition (i), it is reasonable to consider  $f(t)$  to represent a bit stream, i.e. to consider the discretized version of  $f(t)$  - the vector  $f_i$  - to be composed of a set of elements with value 0 or 1. This binary code can of course be based on a key or set of keys which, when reconstructed, is compared to the key(s) for the purpose of authenticating the data. However, this requires the distribution of such keys. Instead, we consider the case where a binary sequence is generated from the signal  $n(t)$ .

### 3.1. Chirp Coding

Given that a binary sequence has been generated from  $n(t)$ , we now consider the method of chirp coding. The purpose of chirp coding is to 'diffuse' each bit over a range of compact support. However, it is necessary to differentiate between 0 and 1 in the sequences. The simplest way to achieve this is to change the polarity of the chirp. Thus, for 1 we apply the chirp  $\sin(\alpha t^2)$ ,  $t \in T$  and for 0 we apply the chirp  $-\sin(\alpha t^2)$ ,  $t \in T$  where  $T$  is the chirp period. The chirps are then concatenated to produce a contiguous stream of data, i.e. a signal composed of  $\pm$ chirps. Thus, the binary sequence 010, for example, is transformed to the signal

$$s(t) = \begin{cases} -\text{chirp}(t), & t \in [0, T); \\ +\text{chirp}(t), & t \in [T, 2T); \\ -\text{chirp}(t), & t \in [2T, 3T). \end{cases}$$

The period over which the chirp is applied depends on the length of the signal to which the watermark is to be applied and the length of the binary sequence. In the example given above the length of the signal is taken to be  $3T$ . In practice, care must be taken over the chirping parameter  $\alpha$  that is applied given a period  $T$  in order to avoid aliasing and in some cases it is of value to apply a logarithmic frequency sweep instead of a linear sweep.

### 3.2. Decoding

Decoding or reconstruction of the binary sequence requires the application of a correlator using the function  $\text{chirp}(t)$ ,  $t \in [0, T)$ . This produces a correlation function that is either -1 or +1 depending upon whether  $-\text{chirp}(t)$  or  $+\text{chirp}(t)$  has been applied respectively. For example, after correlating the chirp coded sequence 010 given above, the correlation function  $c(t)$  becomes

$$c(t) = \begin{cases} -1, & t \in [0, T); \\ +1, & t \in [T, 2T); \\ -1, & t \in [2T, 3T). \end{cases}$$

from which the original sequence 010 is easily inferred - the change in sign of the correlation function identifying a change of bit (from 0 to 1 or from 1 to 0). Note that in practice the correlation function may not be exactly 1 or -1 when reconstruction is undertaken in the presence of additive noise; the binary sequence is effectively recovered by searching the correlation function for changes in sign.

### 3.3. Watermarking

The watermarking process is based on adding the chirp coded data to the signal  $n(t)$ . Let the chirp coded signal be given by the function  $h(t)$ , then the watermarking process is described by the equation

$$s(t) = a \left[ \frac{bh(t)}{\|h(t)\|_\infty} + \frac{n(t)}{\|n(t)\|_\infty} \right]$$

The coefficients  $a > 0$  and  $0 < b < 1$  determine the amplitude and the SNR of  $s$  respectively where

$$a = \|n(t)\|_\infty.$$

The coefficient  $a$  is required to provide a watermarked signal whose amplitude is compatible with the original signal  $n$ . The value of  $b$  is adjusted to provide an output that is acceptable in the application to be considered and to provide a robust reconstruction of the binary sequence by correlating  $s(t)$  with  $\text{chirp}(t)$ ,  $t \in [0, T)$ .

## 4. CODE GENERATION

In the previous section, the method of chirp coding a binary sequence and watermarking the signal  $n(t)$  has been discussed where it is assumed that the sequence is generated from this same signal. In this section, the details of this method are presented. There are a wide variety of coding methods that can be applied [15]. The problem is to convert the salient characteristics of the signal  $n(t)$  into a sequence of bits that is relatively short and conveys information on the signal that is unique to its overall properties. In principle, there are a number of ways of undertaking this. For example, in practice the digital signal  $n_i$  - which is composed of an array of floating point numbers - could be expressed in binary form and each element concatenated to form a contiguous bit stream. However, the length of the code (i.e. the total number of bits in the stream) will tend to be large leading to high computational costs in terms of the application of chirp coding/decoding. What is required, is a process that yields a relatively short binary sequence (when compared with the original signal) that reflects the important properties of the signal in its entirety. Two approaches are considered here: (i) Power Spectral Density decomposition and (ii) Wavelet decomposition [16].

### 4.1. Power Spectral Density Decomposition

Let  $N(\omega)$  be the Fourier transform  $n(t)$  and define the Power Spectrum  $P(\omega)$  as

$$P(\omega) = |N(\omega)|^2$$

An important property of the binary sequence is that it should describe the spectral characteristics of the signal in its entirety. Thus, if, for example, the binary sequence is based on just the low frequency components of the signal, then any distortion of the high frequencies of the watermarked signal will not affect the recovered watermark and the signal will be authenticated. Hence, we consider the case where the power spectrum is segmented into  $N$  components, i.e.

$$\begin{aligned} P_1(\omega) &= P(\omega), & \omega \in [0, \Omega_1) \\ P_2(\omega) &= P(\omega), & \omega \in [\Omega_1, \Omega_2) \\ & \vdots \\ P_N(\omega) &= P(\omega), & \omega \in [\Omega_{N-1}, \Omega_N) \end{aligned}$$

Note that it is assumed that the signal  $n(t)$  is band-limited with a bandwidth of  $\Omega_N$ .

The set of the functions  $P_1, P_2, \dots, P_N$  now represent the complete spectral characteristics of the signal  $n(t)$ . Since each of these functions represents a unique part of the spectrum, we can consider a single measure as an identifier or tag. A natural measure to consider is the energy which is given by the integral of the functions over their frequency range. In particular, we consider the energy values in terms of their contribution to the spectrum as a percentage, i.e.

$$E_1 = \frac{100}{E} \int_0^{\Omega_1} P_1(\omega) d\omega$$

$$E_2 = \frac{100}{E} \int_{\Omega_1}^{\Omega_2} P_2(\omega) d\omega$$

$$\vdots$$

$$E_N = \frac{100}{E} \int_{\Omega_{N-1}}^{\Omega_N} P_N(\omega) d\omega$$

where

$$E = \int_0^{\Omega_N} P(\omega) d\omega.$$

Code generation is then based on the following steps:

1. Rounding to the nearest integer the (floating point) values of  $E_i$  to decimal integer form:

$$e_i = \text{round}(E_i), \quad \forall i$$

2. Decimal integer to binary string conversion:

$$b_i = \text{binary}(e_i)$$

3. Concatenation of the binary string array  $b_i$  to a binary sequence:

$$f_j = \text{cat}(b_i)$$

The watermark  $f_j$  is then chirp coded.

## 4.2. Wavelet decomposition

Wavelet signal analysis is based on convolution type operations which include a scaling property in terms of the amplitude and temporal extent of the convolution kernel (e.g. [3], [17], [18] and [19]). There is a close synergy between the wavelet transform and imaging science. For example, in Fresnel optics, the two-dimensional (coherent) optical wavefield  $u$  generated by an object function  $f$  (in the object plane at a distance  $z$ ) is given by (e.g. [4] and [20])

$$u(x, y, L) = p(x, y, L) \otimes f(x, y)$$

where

$$p(x, y, L) = i \exp\left(i \frac{2\pi z}{\lambda}\right) \frac{1}{L} \exp\left[\frac{i\pi}{L}(x^2 + y^2)\right]$$

and  $L = \lambda z$  for wavelength  $\lambda$ . An important feature of this result is that the amplitude of the kernel  $p$  and its scale length is determined by the reciprocal of the wavelength  $\lambda$ . Physically, this

implies that as the wavelength decreases, the 'resolving power' of an image given by  $I(x, y, L) = |u(x, y, L)|^2$  increases, the bandwidth  $u$  being proportional to  $\lambda^{-1}$ . Thus, by considering a hypothetical Fresnel imaging system, in which the wavelength can be varied by the user, we can consider the imaging system to have multi-resolution properties. The Fresnel transform is essentially a wavelet transform with a wavelet determined by a (two-dimensional) chirp function.

The multi-resolution properties of the wavelet transform have been crucial to their development and success in the analysis and processing of signals. Wavelet transformations play a central role in the study of self-similar or fractal signals. The transform constitutes as natural a tool for the manipulation of self-similar or scale invariant signals as the Fourier transform does for translation invariant signals such as stationary and periodic signals.

In general, the wavelet transformation of a signal  $f(t)$  say

$$f(t) \leftrightarrow F_L(t)$$

is defined in terms of projections of  $f(t)$  onto a family of functions that are all normalized dilations and translations of a prototype 'wavelet' function  $W$ , i.e.

$$\hat{W}[f(t)] = F_L(\tau) = \int f(t) w_L(t, \tau) dt$$

where

$$w_L(t, \tau) = \frac{1}{\sqrt{|L|}} w\left(\frac{\tau - t}{L}\right).$$

The parameters  $L$  and  $\tau$  are continuous dilation and translation parameters respectively, and take on values in the range  $-\infty < L, \tau < \infty, L \neq 0$ . Note that the wavelet transformation is essentially a convolution transform in which  $w(t)$  is the convolution kernel but with a factor  $L$  introduced. The introduction of this factor provides dilation and translation properties into the convolution integral that gives it the ability to analyse signals in a multi-resolution role (the convolution integral is now a function of  $L$ ). A multi-resolution signal analysis is a framework for analysing signals based on isolating variations that occur on different temporal or spatial scales. The basic analysis involves approximating the signal at successively coarser scales through repeated application of a smoothing (convolution) operator.

A necessary and sufficient condition for a wavelet transformation to be invertible is that  $w(t)$  satisfy the *admissibility condition*

$$\int |W(\omega)|^2 |\omega|^{-1} d\omega = C_w < \infty$$

where  $W$  is the wavelets Fourier transform, i.e.

$$W(\omega) = \int w_L(t) \exp(-i\omega t) dt.$$

For any admissible  $w(t)$ , the wavelet transform has an inverse given by [3]

$$f(t) = \hat{W}^{-1}[F_L(\tau)] = \frac{1}{C_w} \int \int F_L(\tau) w_L(t, \tau) L^{-2} dL d\tau.$$

There are a wide variety of wavelets available [i.e. functional forms for  $w_L(t)$ ] which are useful for processing digital signals in 'wavelet space' when applied in discrete form. The properties of the wavelets vary from one application to another but in each

case, the digital signal  $f_i$  is decomposed into a matrix (a set of vectors)  $F_{i,j}$  where  $j$  is the 'level' of the decomposition.

The wavelet transform can be used to generate a suitable code by computing the energies of the wavelet transformation over  $N$  levels. Thus, the signal  $f(t)$  is decomposed into wavelet space to yield the following set of functions:

$$F_{L_1}(\tau), F_{L_2}(\tau), \dots, F_{L_N}(\tau)$$

The (percentage) energies of these functions are then computed, i.e.

$$E_1 = \frac{100}{E} \int |F_{L_1}(\tau)|^2 d\tau$$

$$E_2 = \frac{100}{E} \int |F_{L_2}(\tau)|^2 d\tau$$

⋮

$$E_N = \frac{100}{E} \int |F_{L_N}(\tau)|^2 d\tau$$

where

$$E = \sum_{i=1}^N E_i$$

The method of computing the binary sequence for chirp coding from these energy values follows that described in the method of power spectral segmentation given in previous section.

## 5. CODING AND DECODING PROCESSES

The *Coding* process computes the watermark from the signal and then applies the watermark to the data using wavelet decomposition. The *Decoding* process regenerates the code from the watermarked signal and then recovers the (same or otherwise) code from the watermark. This decoding process provides an error measure based on the result

$$e = \frac{\sum_i |x_i - y_i|}{\sum_i |x_i + y_i|}$$

where  $x_i$  and  $y_i$  are the decimal integer arrays obtained from the input signal and the watermark (or otherwise). Only a specified segment of the data is extracted for watermarking. The segment can be user defined and if required, form the basis for a (private) key system.

### 5.1. Coding process

The coding process is compounded in the following basic steps:

1. Read input.
2. Extract a section of a single vector of the data (note that a WAV contains stereo data, i.e. two vectors arrays).
3. Apply wavelet decomposition using Daubechies wavelets with 7 levels. Note that in addition to wavelet decomposition, the approximation coefficients for the input signal are computed to provide a measure on the global effect of introducing the watermark into the signal. Thus, 8 decomposition vectors in total are generated.
4. Compute the (percentage) 'energy values'.
5. Round to the nearest integer and convert to binary form.

6. Concatenate both the decimal and binary integer arrays.
7. Chirp code the binary sequence.
8. Scale the output and add to the original input signal.
9. Re-scale the watermarked signal.
10. Write output.

### 5.2. Decoding process

The decoding process is as follows:

1. Steps 1-6 in the coding processes are repeated
2. Correlate the data with a chirp identical to that used for chirp coding
3. Extract the binary sequence
4. Convert from binary to decimal
5. Display the original and reconstructed decimal sequence
6. Display the error

Note that in a practical application of this method for authenticating audio files, for example, a threshold can be applied to the error value. If and only if the error lies below this threshold is the data taken to be authentic.

## 6. DISCUSSION

The method of digital watermarking discussed here makes specific use of the chirp function. This function is unique in terms of its properties for reconstructing information (via application of the Matched Filter). The watermark  $f$  extracted from the host signal  $n$  is, in theory, an exact band-limited version of the original watermark.

The approach considered in this paper allows a code to be generated directly from the host signal and that same code used to watermark the signal. The code is therefore self-generating and its reconstruction only requires a correlation process with the watermarked signal to be undertaken. This means that the signal can be authenticated without reference to a known data base. In other words, the method can be seen as a way of authenticating data by extracting a code (the watermark) within a 'code' (the host signal) and is consistent with approaches that attempt to reconstruct information without knowledge of the host data [21].

Audio data watermarking schemes rely on the imperfections of the human audio system. They exploit the fact that the human auditory system is insensitive to small amplitude changes, either in the time or frequency domains, as well as insertion of low amplitude time domain echo's. Spread spectrum techniques augment a low amplitude spreading sequence, which can be detected via correlation techniques. Usually, embedding is performed in high amplitude portions of the signal, either in the time or frequency domains. A common pitfall for both types of watermarking systems is their intolerance to detector de-synchronization and deficiency of adequate methods to address this problem during the decoding process. Although other applications are possible, chirp coding provides a new and novel technique for fragile audio watermarking. In this case, the watermarked signal does not change the perceptual quality of the signal. In order to make the watermark inaudible, the chirp generated is of very low frequency and amplitude. Using audio files with sampling frequencies of over 1000Hz, a logarithmic chirp can be generated in the frequency band of 1-100Hz. Since the human ear has low sensitivity in this band, the

embedded watermark will not be perceptible. Depending upon the band and amplitude of the chirp, the signal-to-watermark (chirp stream) ratio can be in excess of 40dB.

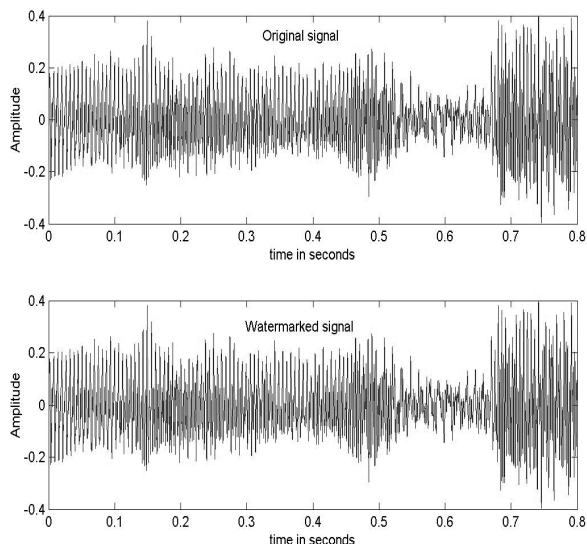


Figure 1: Original signal (above) and chirp based watermarked signal (below).

Figure 1 is an example of an original and a watermarked audio signal which shows no perceptual difference during a listening test. Various forms of attack can be applied which change the distribution of the percentage sub-band energies originally present in the signal including filtering (both low pass and high pass), cropping and lossy compression (MP3 compression) with both constant and variable bit rates. In each case, the signal and/or the watermark is distorted enough to register the fact that the data has been tampered with. An example of this is given in Figure 2 which shows the power spectral density of an original, watermarked and a (band-pass filtered) tampered audio signal. The filtering is such that there is negligible change in the power spectral density. However, the tampering was easily detected by the proposed technique. Finally, chirp coded watermarks are difficult to remove from the host signal since the initial and the final frequency is at the discretion of the user and its position in the data stream can be varied through application of an offset, all such parameters being combined to form a private key.

## 7. EXAMPLE APPLICATION

The proposed scheme has been implemented using MATLAB to provide a watermarking facility for (WAV or MP3) audio files. An example system designed for this purpose is made from:

<http://eleceng.dit.ie/arg/downloads/>

Audio\_Self-Authentication.zip.

After installation of the software, execution of the applications file

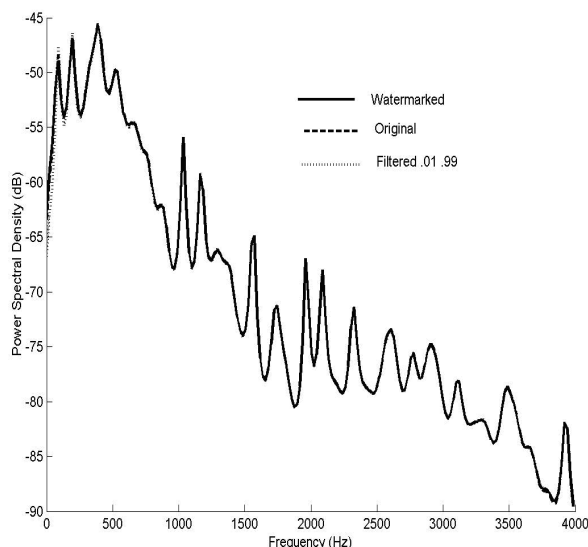


Figure 2: Difference in the power spectral density of the original, watermarked and tampered signal. The tampering has been undertaken using a band pass filter with a normalised lower cut-off frequency of 0.01 and higher cut-off frequency 0.99.

*AudioCode* (contained in the 'Bin' folder) generates the Graphical User Interface (GUI) shown in Figure 3. This GUI provides the user with the following options: Browse for I/O files (WAV or MP3), *Mark* to watermark the data, *Compress* to compress the data to an MP3 file, *Authenticate* to determine whether the data is watermarked (and thereby authentic) or otherwise. There are two principal operations for watermarking an audio file: those associated with a WAV file and those of an MP3 file as discussed below.

### 7.1. Tagging a WAV File

The WAV file is selected (through application of *Browse*) and the name of the output file specified (typically by *Browsing* and then editing the file name as required - the extension is not required). Clicking on the *Mark* button watermarks the file with data derived from the signal via wavelet decomposition and chirp coding. The user has the option of additionally creating an MP3 file of the watermarked audio data by clicking on the *Compress* button.

### 7.2. Tagging a MP3 File

The MP3 file is selected (through application of *Browse*) and the name of the output file specified (typically by *Browsing* and then editing the file name as required - the extension is not required). The system automatically converts the MP3 file to a WAV file for the purpose of watermarking the data. Clicking on the *Mark* button watermarks the file with the data derived from the signal by wavelet decomposition and chirp coding. The user then has the option of re-creating an MP3 file of the watermarked audio data by clicking on the *Compress* button.

### 7.3. Watermark Recovery

The watermarked file is selected through *Browse* as an input file (either a WAV or MP3 file). Clicking the *Authenticate* button executes recovery of the watermark from the signal and regenerates the watermark code by wavelet decomposition of the same (watermarked) signal. If the reconstructed watermark and the regenerated watermark codes match to within a pre-defined tolerance, then the signal is verified as being authentic. If this is not the case, then the system responds with a statement to the effect that the signal has not been authenticated.

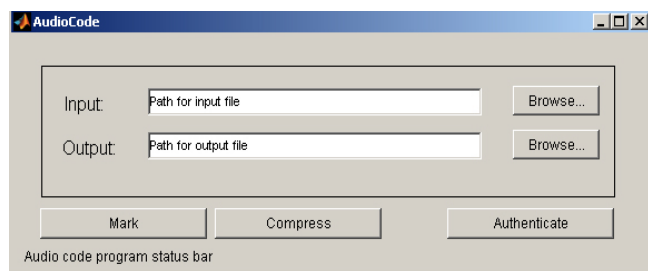


Figure 3: GUI for the audio signal self-authentication system.

## 8. CONCLUDING REMARKS

Chirp coding is generic in the sense that it can be used to watermark any (user defined) bit stream in a signal. For watermarking with plaintexts, the bit stream can be generated using a standard ASCII (7-bit) code. Thus, the use of this method for self-authenticating signals, as discussed in this paper, is just one approach, albeit a useful one. However, in terms of sending and receiving data through some communications channel, the most important feature of chirp coding is the facility it provides for transmitting information through environments with significant amounts of noise, recovery of this information being based on knowledge of the exact chirp function used to 'chirp code'.

The approach proposed in this paper is of specific value for the self-authentication of data for which the method is unique. The proposed scheme has been simulated and tested for various attacks and has been shown to be robust to most attacks with the capability to detect tampering of the signal. This is due to the embedding of a watermark sequence which is derived from the multi-resolution properties of the signal. Objective Difference Grade evaluations using the basic version of Perceptual Evaluation of Audio Quality (PEAQ ITU-R recommendation BS.1387) [22] with ten model output variables was -0.721 which is in the imperceptible range.

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