PASSIVE ADMITTANCE MATRIX MODELING FOR GUITAR SYNTHESIS

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ABSTRACT

In physics-based sound synthesis, it is generally possible to incorporate a mechanical or acoustical immittance (admittance or impedance) in the form of a digital filter. Examples include modeling of the termination of a string or a tube. However, when digital filters are fitted to measured immittance data, care has to be taken that the resulting filter corresponds to a passive mechanical or acoustical system, otherwise the stability of the instrument model is at risk. In previous work, we have presented a simple method for designing and realizing inherently passive scalar admittances, by composing the admittance as a linear combination of positive real (PR) functions with nonnegative weights. In this paper the method is extended to multidimensional admittances (admittance matrices). The admittance matrix is synthesized as a sum of PR scalar transfer functions (second-order filters) multiplied by positive semidefinite matrices. For wave-based modeling, such as digital waveguides (DWGs) or wave digital filters (WDFs), the admittance matrix is converted to a reflectance filter. The filter structure is retained during conversion, resulting in a numerically robust implementation. As an example, a dual-polarization guitar string model based on the DWG approach is connected to the reflectance model parameterized from guitar bridge admittance measurements.

1. INTRODUCTION

In physics-based sound synthesis, the sound of an instrument is generated by modeling the instrument behavior rather than modeling the sound itself. Therefore the model blocks correspond to the main parts of the instrument (for an overview, see [1]). Depending on the modeling paradigm, these models can be parameterized in many ways. For example, it is possible to parameterize parts of the instrument model by a measured mechanical or acoustical immittance (admittance or impedance). As an example, the effect of an immittance (e.g., the instrument bridge) connected to a string is that it changes the modal frequencies and decay times of the string compared to a rigid termination, and provides coupling between the horizontal, vertical, and longitudinal polarizations of the string. Note that we will restrict ourselves to mechanical admittances, but the treatment is equally applicable to other passive (e.g., acoustical) systems and to impedances instead of admittances.

The starting point of such a parameterization is a mechanical admittance measurement of the given part of the instrument (e.g., the bridge). Naturally, all parts of acoustical instruments are passive, that is, they can only dissipate energy that is introduced by the player. In theory, the measured admittance could be directly represented as an FIR or an IIR filter[1] fitted to the measured response. However, often the resulting digital filter does not correspond to a passive termination, that is, at some frequencies it generates power instead of dissipating it. This can happen because of two reasons: the measured impulse response itself may not be passive because of measurement errors, or, due to the fact that the admittance is only approximated by the FIR or IIR filter fitted to the response.

Therefore, instead of straightforward filter design, such a design technique should be used that results in a passive admittance filter. In [2], passive admittance filters are constructed by manually tuning the modal frequencies and decay times of second-order resonators to produce a function similar to the guitar admittance, and a similarly simplified guitar bridge model is presented in [3] by connecting the passive admittance to a scattering junction. In [4], the 2D mechanical admittance of a guitar bridge up to 3 kHz is modeled by a set of mass-spring-damper elements (second-order resonators), and the matrix pencil method is used for parameter estimation. In the frequency-domain guitar model of [5], a standard modal analysis technique (circle fitting) is used up to 1.4 kHz, and above that a random number generator is applied to produce a statistically similar modal behavior as in the measured response. This was necessary because standard modal analysis techniques perform well only in the low frequency region where the modes are separated (up to 1–2 kHz in the case of the guitar), and they cannot easily capture the behavior at high frequencies, where the modal overlap is high.

In [6] we have proposed an admittance filter design method that models the admittance accurately in the low frequency region (up to a few kHz), while at high frequencies, only the general trend of the admittance is modeled. This is motivated by the fact that in sound synthesis, low frequency admittance modeling should be more accurate, since this is the region that influences the decay times of the most important partials of the tone. The nonuniform resolution is achieved by determining the poles of the admittance filter by frequency warped filter design. Then, the admittance transfer function is constructed as a weighted sum of passive (positive real) second-order transfer functions.

This paper extends our previous work [6] to the modeling of admittance matrices. The multidimensional admittance is composed as a linear combination of scalar positive real transfer functions with weighting matrices that are positive semidefinite. The

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[1] It is important to keep in mind that immittance itself is generally not a filter or transfer function but a constraint relation between quantities such as force and velocity, while wave-based reflectance is a filter in the sense of input-output relationship.

admittance matrix is converted to a reflectance matrix filter that retains the parallel second-order filter structure of the admittance formulation.

The organization of this paper is as follows: first, Sec. 2 gives the necessary theoretical background. Then, Sec. 3 proposes the admittance matrix design algorithm, Sec. 4 presents the application of the admittance filter to wave-based modeling, and Sec. 5 gives a guitar bridge modeling example. Finally, Sec. 6 concludes the paper and indicates the areas of future research.

2. BACKGROUND

2.1. Passivity and positive realness

A system is passive if it cannot produce energy. For continuous-time systems, wide literature is available about the subject, as passivity is an important property in network analysis and synthesis as well as in nonlinear control. For passive systems, immittances are positive real (PR) [7].

For rational functions of s that do not have a pole on the closed right-half plane (that is, for asymptotically stable systems), the transfer function matrix $\mathbf{H}(s)$ is PR if and only if

$$\mathrm{Re}\left\{\mathbf{H}(j\omega)\right\} = \frac{1}{2}\left(\mathbf{H}(j\omega) + \mathbf{H}^{\star}(j\omega)\right) \geq 0 \qquad (1)$$

for all real ω [7]. Here * means complex conjugation, and $\mathbf{A} \geq 0$ means that $\mathbf{A}$ is positive semidefinite.

The PR condition for a digital transfer function $\mathbf{H}(z) = \mathbf{H}(e^{-j\vartheta})$ in a rational form with poles in the open unit disk (asymptotically stable systems) is similar to that for the continuous case [8]:

$$\mathrm{Re}\left\{\mathbf{H}(e^{-j\vartheta})\right\} = \frac{1}{2}\left(\mathbf{H}(e^{-j\vartheta}) + \mathbf{H}^{\star}(e^{-j\vartheta})\right) \geq 0. \qquad (2)$$

That is, it is enough to check positive realness on the unit circle, by looking at the frequency response. Functions satisfying Eq. (2) are called "circle positive real" in [8]. However, we will use "positive real" both for discrete-time and continuous-time transfer functions in this paper.

Fitting positive real functions to measurement data are frequently used in modeling and verification of integrated circuits, therefore, a wide range of continuous-time methods are available (see, e.g., [9, 10]). Most probably these sophisticated algorithms could be modified for discrete-time systems. However, they did not find their way to the musical acoustics and sound synthesis community, probably due to their complexity. In addition, the modal framework (outlined in Sec. 2.2) also provides passive models and it is better related to the physical structure of the instrument.

2.2. Modal framework

The quest for a PR transfer function can be simplified if some assumptions are made on the structure. First, let us define the admittance matrix $\mathbf{Y}$:

$$\mathbf{v} = \mathbf{Y}\mathbf{f} \qquad (3)$$

where $\mathbf{f} = [F_1, \ldots, F_K]^T$ is a column vector composed of the forces exciting the structure at positions $1, \ldots, K$, and $\mathbf{v} = [v_1, \ldots, v_K]^T$ is a column vector composed of the velocities of points $1, \ldots, K$.

In modal analysis, the general assumption is that the structure can be described as a set of masses that are connected by linear springs and linear dampers [11]. Then, the vibration of the structure can be decomposed to a sum of R normal modes with different modal frequencies ω_r, decay rates σ_r and modal shapes $\mathbf{\Phi}_r$. It is a common assumption in modal analysis that the damping is viscous and it is distributed proportionally to the mass and stiffness elements, referred as proportional damping in the literature. In this case the modal shapes $\mathbf{\Phi}_r$ are real and the mechanical admittance (mobility) matrix of the system can be written as [12]

$$\mathbf{Y}(j\omega) = \sum_{r=1}^{R} \mathbf{\Phi}_r^T \mathbf{\Phi}_r \frac{j\omega}{m_r(\omega_r^2 - \omega^2 + 2j\sigma_r\omega_r\omega)} \qquad (4)$$

where m_r is the effective mass of mode r, and $\mathbf{\Phi}_r^T\mathbf{\Phi}_r$ is a rank 1 size K square matrix which is positive semidefinite, since the elements of $\mathbf{\Phi}_r$ are real (not complex). The scalar transfer functions in Eq. (4) are PR because their phase span from $-\pi/2$ to $\pi/2$. Thus, the real part of $\mathbf{Y}$ will be positive semidefinite for all ω frequencies, since $\mathbf{Y}$ is a linear combination of real positive semidefinite matrices $\mathbf{\Phi}_r^T\mathbf{\Phi}_r$ with positive real weights.

A straightforward approach for modeling a given (measured) admittance is to use standard modal analysis tools to fit a modal model of Eq. (4) to the measured data, and implement a discretized version of Eq. (4). However, there are two related problems which prevent us from doing so. First, standard modal analysis techniques work only in such regions of the transfer function, where the modal overlap is low (modes are well separated). Therefore, accurate modal parameters could be obtained for the low frequency region of instrument bridges only. In addition, in the case of sound synthesis applications, the model order is significantly smaller compared to the order of the system, which means that the assumptions used to derive Eq. (4) are no longer true. For example, the poles of the model do not necessarily correspond to the poles of the system, and the "modal shapes" of the model should approximate the gross behavior of all the system modes having modal frequencies near to the corresponding pole frequency of the model.

3. THE PASSIVE ADMITTANCE MODEL

Here we propose using a modification of the modal model by interchanging the $\mathbf{\Phi}_r^T\mathbf{\Phi}_r$ rank 1 matrices with general (full rank) symmetric $\mathbf{Y}_r$ matrices, giving more degrees of freedom in modeling. This actually corresponds to allowing maximum K modal shapes for each pole-pair of the model instead of a single mode. As a result, the admittance is modeled as

$$\mathbf{Y}(z) = \sum_{r=1}^{R} \mathbf{Y}_r H_r(z) \qquad (5a)$$

$$H_r(z) = \frac{1 - z^{-2}}{(1 - p_r z^{-1})(1 - p_r^{\star} z^{-1})}, \qquad (5b)$$

where $H_r(z)$ are the bilinearly transformed discrete-time versions of the second-order functions of Eq. (4). If a positive real function $H(s)$ is converted to a discrete-time function $H(z)$ by the bilinear transform, it remains positive real [2, 13]. Therefore, $H_r(z)$ are PR. A sufficient condition for the admittance model $\mathbf{Y}(z)$ to be PR is that all the $\mathbf{Y}_r$ matrices are positive semidefinite, because in this case we have

$$\mathrm{Re}\{\mathbf{Y}(z)\} = \mathrm{Re}\{\sum_{r=1}^{R} \mathbf{Y}_r H_r(z)\} = \sum_{r=1}^{R} \mathbf{Y}_r \mathrm{Re}\{H_r(z)\} \geq 0, \qquad (6)$$

since the linear combination of positive semidefinite matrices $\mathbf{Y}_r$ with nonnegative scalar weights $\mathrm{Re}\{H_r(z)\}$ is also positive semidefinite.

3.1. Parameter estimation

The parameters of the admittance model Eq. (5) are obtained from a measured admittance matrix $\mathbf{Y}_\mathrm{m}(z)$ as follows:

1. **Pole positioning**: The measured admittance $\mathbf{Y}_\mathrm{m}(z)$ contains K^2 transfer functions (or, impulse responses, if the data is available in the time domain), of which $K(K+1)/2$ are independent, due to symmetry. The task is to find a common-denominator model that best describes all the $K(K+1)/2$ transfer functions, since the poles are the same for each transfer function in the model of Eq. (5). This can be done by various common-denominator algorithms used in modal analysis. Here we are using a common-denominator version of autoregressive modeling (or, equivalently, linear prediction) in the time domain, that results in an all-pole model. As a notation, let us define $\mathbf{Y}[n]$ as the element-wise inverse z transform of $\mathbf{Y}(z)$, which is actually the impulse response of the admittance matrix. Accordingly, $\mathbf{Y}_\mathrm{m}[n]$ is the measured admittance impulse response. Then, the regression error for the $Y_{ij,\mathrm{m}}[n]$ element of the matrix $\mathbf{Y}_\mathrm{m}[n]$ can be written as

$$E_{ij} = \sum_{n=L}^{N} \left(Y_{ij,\mathrm{m}}[n] + \sum_{l=1}^{L} a_m Y_{ij,\mathrm{m}}[n-l] \right)^2, \quad (7)$$

where L is the order of the denominator, and N is the length of the measured admittance impulse response $Y_{ij,\mathrm{m}}[n]$. Note that the denominator coefficients a_m are the same for all the ij elements in Eq. (7) and the task is to find this common set of a_m coefficients such that the total error

$$e = \sum_{i=1}^{K} \sum_{j=1}^{i} E_{ij} \quad (8)$$

is minimal. This is a linear least-squares problem that is solved by the normal equations in a closed form. Note that the index j in the second sum of Eq. (8) runs to i instead of K because it is sufficient to compute the error for the lower triangular part $j \le i$ of $\mathbf{Y}_\mathrm{m}$ only, since $\mathbf{Y}_\mathrm{m}$ is symmetric.

As already stated in the Introduction, our goal is to model the admittance more precisely at low frequencies compared to the high ones. This has to be reflected by resolution of pole positioning, since the poles determine the frequency resolution of the design, similarly to Kautz [14] and parallel filters [15]. Therefore, the above common-denominator model is estimated in the warped domain [16]. For that, all the measured impulse responses are frequency warped with parameter λ, and the common-denominator autoregressive model is estimated based on this warped data. Then, the roots of the denominator $\tilde{p}_r$ are found and "dewarped" by the expression

$$p_r = \frac{\tilde{p}_r + \lambda}{1 + \tilde{p}_r} \quad (9)$$

The poles p_r are used for constructing the second-order functions $H_r(z)$ according to Eq. (5b).

2. **Weight matrix estimation**: The final step is to estimate the weight matrices $\mathbf{Y}_r$, which is a linear-in-parameter problem with the positive-semidefiniteness constraints $\mathbf{Y}_r \ge 0$. The time-domain error for one matrix element is

$$E'_{ij} = \sum_{n=0}^{N} \left(Y_{ij}^r h_r[n] - Y_{ij,\mathrm{m}}[n] \right)^2 \quad (10)$$

where Y_{ij}^r is the ij element of $\mathbf{Y}_r$ (thus, the superscript r is not a power but an index), and $h_r[n]$ is the inverse z transform of $H_r(z)$.

The optimal set of parameters $\mathbf{Y}_r$ are obtained by solving

$$\mathrm{minimize} \quad e' = \sum_{i=1}^{K} \sum_{j=1}^{i} E'_{ij} \quad (11a)$$

$$\text{subject to} \quad \mathbf{Y}_r \ge 0. \quad (11b)$$

We propose a relatively simple (although probably suboptimal) solution to Eq. (11). First, we find the $\mathbf{Y}_r$ matrices without the constraint of Eq. (11b). Since now the elements of $\mathbf{Y}_r$ become independent, the total error is minimal if all E'_{ij} are minimal. Thus, the problem reduces to minimizing Eq. (10) for all E'_{ij} independently, which are separate linear least-squares problems with a closed-form solution. Then, the resulting $\mathbf{Y}_r$ matrices are "converted" to positive semidefinite matrices. This last step involves finding the nearest positive semidefinite matrix to $\mathbf{Y}_r$, which is similar to finding the nearest valid correlation matrix [17] without the unit diagonal constraint. This is achieved by computing the spectral decomposition of $\mathbf{Y}_r$, discarding the negative eigenvalues and their eigenvectors, and reconstructing the matrix from the remaining positive eigenvalues and corresponding eigenvectors.

This basic procedure is slightly improved if the diagonal elements E'_{ii} in Eq. (10) are minimized with the nonegativity constraints $Y_{ii}^r \ge 0$, which is a standard nonnegative least squares problem. This improves the results because the nonnegativity of the diagonal elements of a matrix is a necessary condition for positive semidefiniteness.

3.2. Implementation

The admittance model of Eq. (5) corresponds to a K input K output MIMO filter which can be straightforwardly implemented by K^2 independent transfer functions. However, by noting that the transfer functions have common poles, a more efficient implementation structure is obtained. Inserting Eq. (5a) into Eq. (3) gives

$$\mathbf{v} = \left[\sum_{r=1}^{R} \mathbf{Y}_r H_r(z) \right] \mathbf{f} = \sum_{r=1}^{R} \mathbf{Y}_r \left[H_r(z) \mathbf{f} \right]. \quad (12)$$

For each mode of the model, the K input signals (forces F_k acting on positions $1, \ldots, K$) are filtered by the second-order filter $H_r(z)$, giving K intermediate signals. Then, the vector formed from these signals is multiplied by $\mathbf{Y}_r$, leading to the velocity contribution $\mathbf{v}_r = [v_1^r, \ldots, v_K^r]$ of mode r. This is done for all the modes $1, \ldots, R$, and the results $\mathbf{v}_1, \ldots \mathbf{v}_R$ are summed to give the velocity vector $\mathbf{v}$. This is shown in Fig. 1 for a single mode r, implementing one term $\mathbf{Y}_r[H_r(z)\mathbf{f}]$ of the sum of Eq. (12).

The computational complexity of the "admittance filtering" for a K by K admittance matrix with R modes is composed as the following:

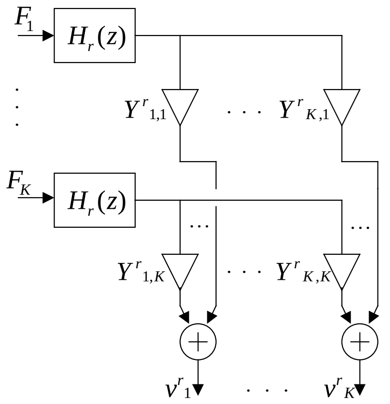

Figure 1: *Implementation of the admittance matrix: the block scheme of one term* $\mathbf{Y}_r[H_r(z)\mathbf{f}]$ *of the sum of Eq. (12).*

1. RK second-order filters $(1-z^{-2})/[(1-p_r z^{-1})(1-p_r^{\star} z^{-1})]$ leading to $2RK$ multiplications and $3RK$ additions
2. R multiplications with matrices $\mathbf{Y}_r$ requiring RK^2 multiplications and $R(K-1)K$ additions
3. summing the velocity contributions of the R modes meaning $(R-1)K$ additions

Roughly, the required computational power is $RK(K+3)$ multiply and accumulate instructions per sample.

4. CONVERSION TO A REFLECTANCE MATRIX

A passive admittance function gives the relation between force and velocity for a mechanical system. It seems that it could therefore be directly applied as a termination of a finite-difference string model, where the force acting on the termination is computed by the string, then this force is filtered by the admittance form as a filter giving the velocity of the termination, which is used in the string model for the next iteration. However, interconnecting passive elements in such a way often results in unstable systems, unless special measures are taken to ensure numerical energy conservation [18].

This problem is automatically avoided in wave-based modeling [18], when the admittance is formulated as a function of wave variables instead of the Kirchhoff variables. In this case, it will be a reflectance filter producing a reflected wave to an incident wave (see the footnote in Introduction). The following derivations for converting the admittance formulation to a reflectance filter are similar to that for the scalar admittance case [6].

4.1. Vector-waveguide termination

Digital waveguide modeling is the most efficient paradigm for modeling the 1-D wave equation. It is based on spatial and temporal discretization of the traveling wave solution for the wave equation [2]. In its basic form, the digital waveguide models the wave propagation in one polarization. However, the idea can be extended to multivariable waveguides so that the elements in the delay line are vectors instead of scalars [19]. For example, for a dual-polarization guitar string model, each delay element contains two variables, giving the string displacement perpendicular and parallel to the guitar body. These multivariable waveguides will be called vector-waveguides in the rest of the paper.

Here we derive a reflectance filter for the case where a multiple-polarization single string (implemented by a vector-waveguide) with a characteristic admittance matrix $\mathbf{Y}_0$ is connected to a termination having an admittance matrix $\mathbf{Y}(z)$. Note that for the case of a linear string $\mathbf{Y}_0$ is diagonal, since there is no internal coupling between the polarizations.

Similarly to the one-dimensional case [2], the reflected velocity wave vector $\mathbf{v}^-$ is obtained from the incident wave vector $\mathbf{v}^+$ as

$$\mathbf{v}^- = \mathbf{H}_{\mathrm{v}}(z)\mathbf{v}^+ = (\mathbf{Y}(z)+\mathbf{Y}_0)^{-1}(\mathbf{Y}(z)-\mathbf{Y}_0)\mathbf{v}^+ \qquad (13)$$

where $\mathbf{H}_{\mathrm{v}}(z)$ is the reflectance matrix for velocity waves.

In theory, the parameters of $\mathbf{H}_{\mathrm{v}}(z)$ could be computed by inserting Eq. (5) into Eq. (13) and rearranging the matrix elements to a rational form, but this would be a very tedious and numerically badly conditioned task. In addition, the physically meaningful modal-like filter structure of Eq. (5) would be lost.

Therefore, we suggest constructing the reflectance filter in such a way that preserves the parallel structure of the admittance formulation. First, the admittance form is decomposed to the immediate response $\mathbf{Y}_{\mathrm{i}}$ (which equals to the first sample $\mathbf{Y}[0]$ of the admittance impulse response) and to the response which depends only on past inputs $z^{-1}\mathbf{Y}_{\mathrm{p}}(z)$ (where $\mathbf{Y}_{\mathrm{p}}(z)$ is the z transform of $\mathbf{Y}[n-1]$ with $n \geq 1$), giving

$$\mathbf{Y}(z) = \mathbf{Y}_{\mathrm{i}} + z^{-1}\mathbf{Y}_{\mathrm{p}}(z). \qquad (14)$$

The decomposition can be done for the second-order filters $H_r(z)$ of Eq. (5) separately [20]:

$$H_r(z) = \frac{1-z^{-2}}{1+a_{r,1}z^{-1}+a_{r,2}z^{-2}} =$$
$$= 1 + z^{-1}\frac{b_{r,1}+b_{r,2}z^{-1}}{1+a_{r,1}z^{-1}+a_{r,2}z^{-2}} = 1 + z^{-1}H_{\mathrm{p},r}(z) \qquad (15)$$

with $b_{r,1} = -a_{r,1}$ and $b_{r,2} = -1 - a_{r,2}$ for $r = 1,\ldots,R$. Thus, the two parts of the admittance filter become

$$\mathbf{Y}_{\mathrm{p}}(z) = \sum_{r=1}^{R}\mathbf{Y}_r H_{\mathrm{p},r}(z) \qquad (16a)$$

$$\mathbf{Y}_{\mathrm{i}} = \sum_{r=1}^{R}\mathbf{Y}_r. \qquad (16b)$$

Note that $\mathbf{Y}_{\mathrm{p}}(z)$ has the same structure as $\mathbf{Y}(z)$ of Eq. (5a), the only difference is that $H_r(z)$ are exchanged for the filters $H_{\mathrm{p},r}(z)$ having different numerator coefficients. Therefore, the filtering computation is done in the same way as explained in Sec. 3.2.

Then, substituting Eq. (14) into Eq. (13) yields the formula for computing the reflected velocity vector:

$$\mathbf{v}^- = (\mathbf{Y}_{\mathrm{i}}+\mathbf{Y}_0)^{-1}\left[z^{-1}\mathbf{Y}_{\mathrm{p}}(z)(\mathbf{v}^+ - \mathbf{v}^-) + (\mathbf{Y}_{\mathrm{i}}-\mathbf{Y}_0)\mathbf{v}^+\right]. \qquad (17)$$

This is illustrated in Fig. 2. The non-computable delay-free loop is avoided because of the decomposition to $\mathbf{Y}_{\mathrm{i}}$ and $\mathbf{Y}_{\mathrm{p}}(z)$, leading to the z^{-1} terms in Fig. 2. Note also that instead of the inversion of a frequency-dependent matrix $(\mathbf{Y}(z)+\mathbf{Y}_0)$ as in Eq. (13) only a matrix with constant elements $(\mathbf{Y}_{\mathrm{i}}+\mathbf{Y}_0)$ has to be inverted. The model of Fig. 2 can be directly used to model the effects of a

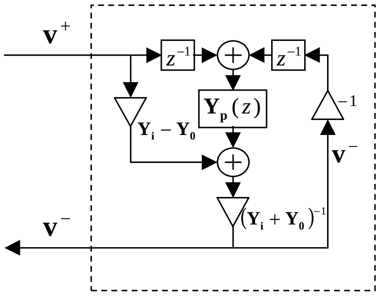

Figure 2: *The reflectance matrix filter.*

multidimensional admittance by connecting the multivariable delay lines of a vector-waveguide to its input $\mathbf{v}^+$ and output $\mathbf{v}^-$. In addition to the delay lines, the vector-waveguide string model should also incorporate loop filters that model string losses and dispersion, and fractional-delay filters for tuning [2].

4.2. Wave-digital filter formulation

Similar, but more tedious derivations can be performed for the case when more multiple polarization strings (more vector-waveguides) are connected to the same termination. However, that case is handled in a more flexible way by constructing a wave-digital filter (WDF) [1, 2] admittance element and connecting it to the vector-waveguide string models by parallel adaptors. For computability reasons, the WDF waveport has to be made free of immediate reflection. This can be easily done since the WDF formulation is independent of (waveguide) impedance(s) connected to it through adaptors, therefore, $\mathbf{Y}_0$ can be chosen freely. The immediate reflection is avoided by setting $\mathbf{Y}_0 = \mathbf{Y}_\mathrm{i}$ (see Fig. 2).

5. GUITAR BRIDGE ADMITTANCE MODELING

We have measured the two-dimensional admittance of an acoustic guitar bridge (Gibson, from 1960's) near the lowest (E) string. The bridge was excited by the wire breaking technique [5] and the movement of the bridge was measured by a miniature accelerometer. The basic idea of the method is that a wire is thread around the string near the bridge and pulled by hand with an increasing force. When the wire breaks, the static force disappears, corresponding to a step excitation plus a constant DC force (the latter has no effect if the system is linear). Since the wire-breaking technique gives the admittance step response if velocity is measured, measuring the acceleration gives the admittance impulse response directly.

The bridge was excited with the wire breaking in the direction perpendicular (y direction) and parallel to the body (z direction). The acceleration was also measured in these two directions. This gave a 2 by 2 admittance impulse response matrix

$$\mathbf{Y}_\mathrm{m}[n] = \begin{bmatrix} Y_{yy,\mathrm{m}}[n] & Y_{yz,\mathrm{m}}[n] \\ Y_{zy,\mathrm{m}}[n] & Y_{zz,\mathrm{m}}[n] \end{bmatrix} \tag{18}$$

where the "m" subscript indicates that these are measured values, which are then approximated by the admittance model impulse response $\mathbf{Y}[n]$. Note that in theory $Y_{yz,\mathrm{m}}[n] = Y_{zy,\mathrm{m}}[n]$, but there are always some differences due to measurement errors. However, for model fitting, a symmetric $\mathbf{Y}_\mathrm{m}[n]$ matrix is assumed (see

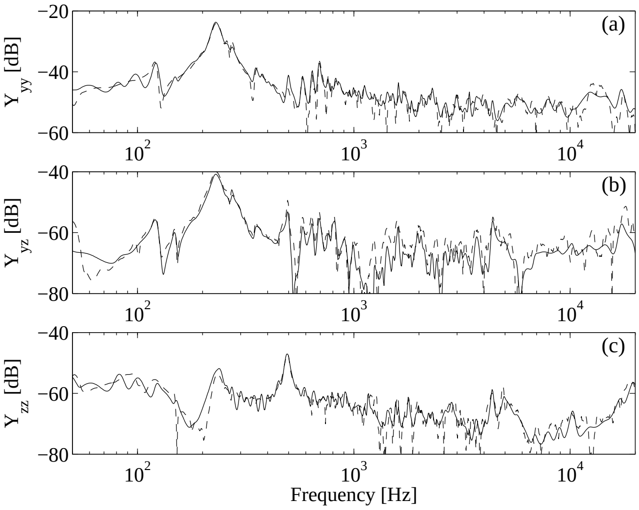

Figure 3: *Modeling a measured guitar bridge admittance by the proposed passive admittance model with 100 second-order filters for the (a) yy, (b) yz, and (c) zz elements of the admittance matrix. Dashed line: measured, solid line: modeled responses.*

Sec. 3.1). This is most easily satisfied by using only one of the two responses (e.g., the less noisy one).

The parameters of the admittance model were estimated in the time-domain by the parameter estimation procedure outlined in Sec. 3.1. The results of the parameter estimation for an admittance model having 100 second-order filters ($R = 100$) are shown in Fig. 3. Naturally, the accuracy of the fit can be increased by using higher filter orders, but for sound synthesis, even using $R = 100$ second-order sections is an overkill, and it is only shown to demonstrate the accuracy of the design. The transfer functions of a more practical admittance model with 30 second-order filters are displayed in Fig. 4. We have found that filter orders in this range provide a good compromise between sound quality and computational efficiency.

Then, the admittance matrix formulation is converted to a 2D reflectance filter as described in Sec. 4.1. A synthesized example when a 2D vector-waveguide corresponding to the lowest string of the guitar ($f_0 = 82$ Hz) is connected to the 2D reflectance filter with $R = 30$ is displayed in Fig. 5. The vector-waveguide includes one-pole lowpass filters [2] to model the losses of the string. The string model is excited by a triangle-shaped initial displacement in the z direction, which approximates a pluck excitation parallel to the guitar body. The output of the model is the bridge velocity in the y direction (perpendicular to the body). Figure 5 shows the amplitude envelopes of the first six partials. It can be seen that strong beating and two-stage decay appears for those partials which are in that frequency range where the elements of the admittance matrix are large. For example, partial No. 3 with $f_3 = 246$ Hz is around the main peak of the guitar admittance (See Fig. 4).

6. CONCLUSION

This paper has presented a methodology for constructing inherently passive admittance matrix models from measured admittances. The admittance matrix is synthesized as a sum of positive semidef-

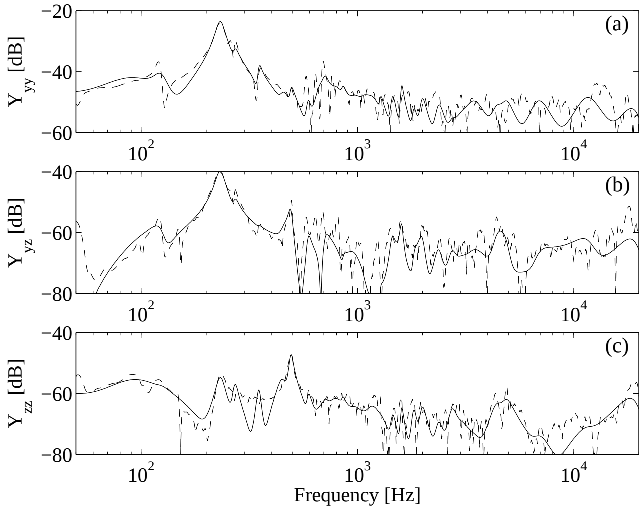

Figure 4: *Modeling a measured guitar bridge admittance by the proposed passive admittance model with 30 second-order filters for the (a) yy, (b) yz, and (c) zz elements of the admittance matrix. Dashed line: measured, solid line: modeled responses.*

inite matrices $\mathbf{Y}_r$ multiplied by scalar positive-real transfer functions $H_r(z)$. The $H_r(z)$ transfer functions are implemented by second-order IIR filters. The poles of the transfer functions are estimated in the frequency-warped domain, giving more emphasis to the low frequency region of the measured admittance functions. For parameter estimation, a relatively simple method was presented that first obtains the elements of $\mathbf{Y}_r$ separately by least squares optimization and then adjusts $\mathbf{Y}_r$ matrices so that they become positive semidefinite.

For wave-based modeling (such as digital waveguides or wave digital filters), the admittance matrix model is converted to a reflectance matrix in such a way that the parallel filter structure is retained, resulting in a numerically robust implementation. As an example, the paper presented a 2D admittance matrix model based on guitar bridge measurements, and showed the results when the obtained reflectance matrix model is connected to a digital-waveguide based dual-polarization string model. Note that the synthesis model remains stable even if the admittance matrix is estimated from less-than-optimal (e.g., noisy or distorted) measurements, since the passivity of the admittance model is guaranteed by the proposed method.

Future research may include the development of improved parameter estimation techniques for the admittance model, and the extension of the current dual-polarization single-string guitar model to a version where all the six strings are coupled at the termination. Moreover, it is believed that the methodology could be applied in other fields besides sound synthesis. For example, the joint vibration of two connected structures could be robustly simulated by describing the objects with their reflectance matrices (obtained from their admittance models) and connecting them with wave digital filter adaptors.

Sound examples are available at
`http://www.mit.bme.hu/~bank/publist/dafx10adm`.

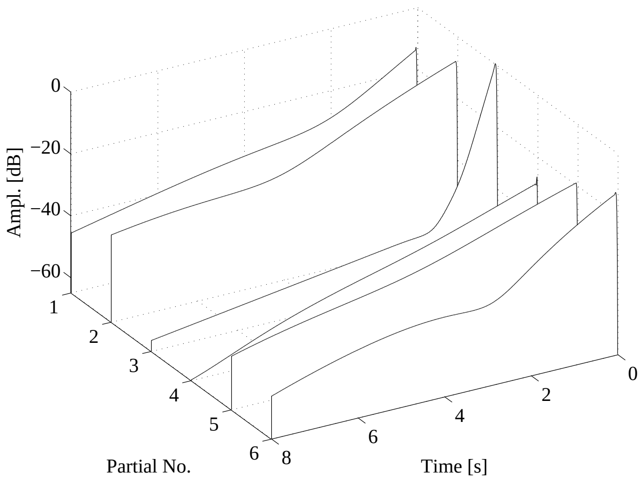

Figure 5: *Partial envelopes of a synthesized guitar sound generated by a 2D vector-waveguide connected to the reflectance matrix with $R = 30$ second-order sections.*

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