# GRAPHIC EQUALIZERS BASED ON LIMITED ACTION NETWORKS

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## ABSTRACT

Several classic graphic equalizers, such as the Altec 9062A and the "Motown EQ," have stepped gain controls and "proportional bandwidth" and used passive, constant-resistance, RLC circuit designs based on "limited-action networks." These are related to bridged-T-network EQs, with several differences that cause important practical improvements, also affecting their sound. We study these networks, giving their circuit topologies, design principles, and design equations, which appear not to have been published before. We make a Wave Digital Filter which can model either device or an idealized "Exact" version, to which we can add various new extensions and features.


Figure 1: *Altec 9062A (top) and Motown EQ serial # 75 (bottom). (Not to scale–The Motown is actually significantly wider.)*

## 1. INTRODUCTION

A family (Fig. 1) of classic 1960s graphic equalizers (EQ), including the Altec 9062A ("the 9062A"), one of the first ever designed, and the Motown EQ ("the Motown"), custom-built in-house by Motown Records employees, are based on passive RLC (resistors, capacitors, and inductors) constant-resistance circuits.

A main concept used in passive EQs [1–9] is that they use "impedance matching"; their input and output impedances are equal to a common line impedance, $R_0$, commonly $600\,\Omega$. A passive EQ with proper impedance matching can be made by cascading blocks that each have the *same* input and output impedance. Obeying this "constant-resistance" principle [3, 4, 10] leads to an overall transmission (or insertion loss) that is equal to the product of the transmissions (resp., insertion losses) of each block. E.g., the Altec 9067B, Allison Laboratories 2AB filter, and RCA Mark II Sound Effects Filter [11] cascade highpass section(s) with lowpass section(s) to create a bandpass filter. Cascading multiple boost/cut filters creates a graphic EQ. This approach, with some tricks we will describe, is used in the 9062A/Motown.

In constant-resistance designs, pairs of impedances $Z_1$ and $Z_2$ must satisfy the constant-resistance property by obeying $Z_1 Z_2 = R_0^2$. They are said to be "inverse with respect to $R_0$" [3, 10].

We study a specific constant-resistance network EQ used in the 9062A/Motown from several perspectives, including (a) using board traces and advertising materials to reverse engineer the circuit topologies, (b) reviewing commonalities and differences to well-known EQs, (c) using electrical measurement to understand their magnitude responses, and (d) using math to derive its hitherto unpublished design equations. Due to a lack of published schematics and some extant misinformation, these EQs are somewhat misunderstood. We aim to correct the record.



The paper is structured as follows. In §2 we review extant information on the 9062A/Motown and give some preliminaries. In §3 we give design procedures for resistors. In §4 we give design procedures for the reactances. In §5 we comment on the multiband "trick" devised by the 9062A designers. In §6 we briefly describe a digital model. §7 concludes and dicusses some extensions.

## 2. REVIEW / PRELIMINARIES

The 9062A was designed by Arthur C. Davis in the 1960s [7, 13], and the Motown uses the same EQ design[1], as reported by former Motown head of engineering Michael McLean [14]. Since there were very few Motown EQs made (according to McLean, only 46 [14], although the device owned by Soundtoys, Inc. curiously has serial number 75), we can focus on the 9062A.

The 9062A's advertising materials [12] states that the circuit is a "Bridged-'T', constant 'K' " network, with input and output impedance of $600\,\Omega$, a flat insertion loss of $16\,\mathrm{dB}$, that it has seven frequency bands centered at 50, 130, 320, 800, 2000, 5000, and $12{,}500\,\mathrm{Hz}$[2], and gain controls in $1\,\mathrm{dB}$ increments, from $-8$ to $+8\,\mathrm{dB}$ (which we could also see on the front panel of the device).[3] A family of magnitude responses show each band's $\pm 8\,\mathrm{dB}$ setting. Fig. 2 shows these magnitude responses, extracted using WebPlotDigitizer [15]. We measure the bandwidth of each trace by looking at where it crosses $\pm 4\,\mathrm{dB}$.[4] The bandwidths vary from

[1] It also includes built-in active makeup amplifier (with a dc blocking capacitor), input and output transformers, and an overall gain control. Modeling and analyzing these elements is beyond the scope of this article.

[2] In both [12] and [8], the $130\,\mathrm{Hz}$ band is curiously labeled as $128\,\mathrm{Hz}$.

[3] An article that discusses the 9062A [7] claims that "The operator's imagination is about the only limitation the instrument imposes," although more precisely there are $17^7 = 410,338,673$ possible configurations.

[4] To increase accuracy, we use linear interpolation at the band edges and

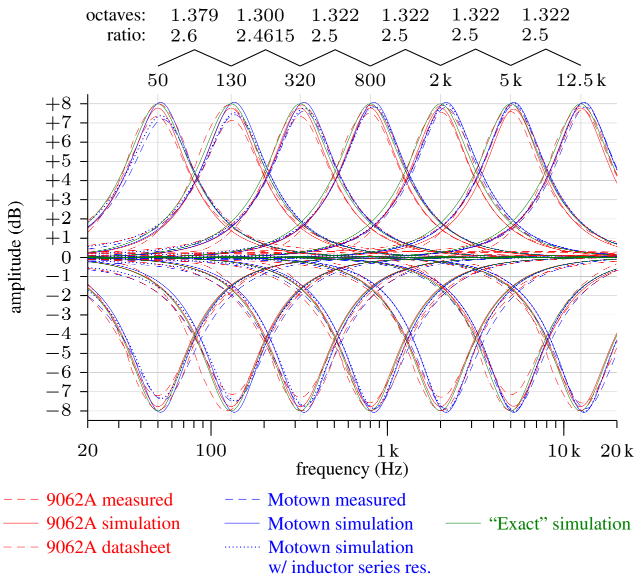



Figure 2: *Magnitude response curves for each individual band at* $\pm 8\,\text{dB}$*, with all others at* $0\,\text{dB}$*. As in [12], throughout the paper, we show magnitude response with respect to the flat insertion loss.*

1.6643 to 1.6994 (i.e., from 0.7349 to 0.7651 octaves) with an average of 1.6803 (0.7487 octaves).

[7] repeats many of these details, mentioning that the center frequencies are "approximately 1½ octaves apart." Fig. 2 shows the actual spacings between bands; the ratio between adjacent center frequencies varies between 2.4615 and 2.6 (1.3 to 1.379), and is 2.5 for most bands (1.322 octaves).

The venerable Audio Cyclopedia [8, pp. 312–313] briefly discusses the 9062A, repeating some of this information and stating "The basic design for the equalizer to be described is the constant-B attenuator, described in Question 6.124," referring to an earlier passage [8, pp. 309–311] that describes Miller and Kimball's constant-B ("constant-bandwidth") EQ design [5].

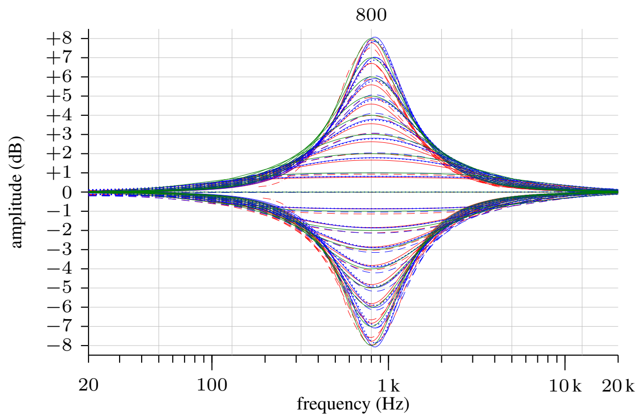
Constant-B EQs, like all passive EQs, use LC tanks as frequency selective elements, and have the special property that, as the peak gain is changed, the bandwidth remains constant without changing the reactances. A boost/cut constant-B EQ needs two sets of tanks, one for boost and one for cut, but the same set of tanks can be used for both if the maximum loss is $\approx 8.36\,\text{dB}$ [8]. This allows constant-bandwidth to be achieved with many fewer reactances than earlier designs, but with more variable resistors.

We appear to know quite a bit about the 9062A. Although one might see something suspicious in the insertion loss claim[5], it feels that a path forwards is emerging. Estimating each band's bandwidth, plugging it, control gains, and band frequencies into known design equations, and hoping that the suspicious claim somehow resolves, perhaps we will get a picture of the EQ's design.

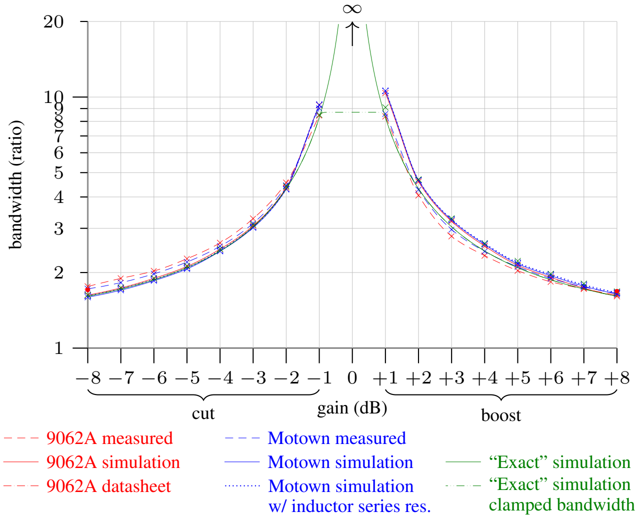
However, there is one gigantic problem. Claims aside, the circuit design used in the Altec 9062A and the Motown EQ is *NOT a constant-B design*! We can see that this is the case in several ways.

parabolic interpolation over the peaks [16].

[5]If the circuit has 7 bands, each of which would have 8 dB of insertion loss in a conventional design (due to the maximum boost/cut of $\pm 8\,\text{dB}$), then how is the total insertion loss only 16 dB, rather than $7 \times 8 = 56\,\text{dB}$?




(a) *Magnitude response curves from sweeping the* 800 Hz *band from* $-8$ *to* $+8\,\text{dB}$ *in* 1 dB *increments, with the rest at* 0 dB.




(b) *The "proportional-bandwidth" relationship between boost/cut and bandwidth. Other than for the "exact" simulations, the lines are just for legibility—the original devices only had stepped (integer) gains.*

Figure 3: *Studying the* 800 Hz *band. Legend applies to both, except datasheet trace is (a) only and clamped bandwidth is (b) only.*

First, we examine magnitude responses[6]. Fig. 3a shows a sweep of the 800 Hz band. The bandwidth changes with the gain, increasing the further the band is from flat. Fig. 3b shows the relationship between gain and bandwidth—-neither the 9062A nor Motown are "constant bandwidth," since these lines are not horizontal. Fig. 4 shows the magnitude responses from moving all the gain sliders in tandem. Due to the proportional-bandwidth behavior, the shape is dramatically inconsistent. Although the gain for an individual band in isolation matches the control gains closely, there is a huge discrepancy as soon as multiple bands are moved together—a huge part of the character of the device.

Second, we examine the schematics. Neither the 9062A nor Motown have published schematics, so we manually traced both circuits. Fig. 5a shows a simplified (one band, most component values suppressed) schematic in boost mode. It is very different

[6]measured from the real devices with a Prism Sound dScope Series III

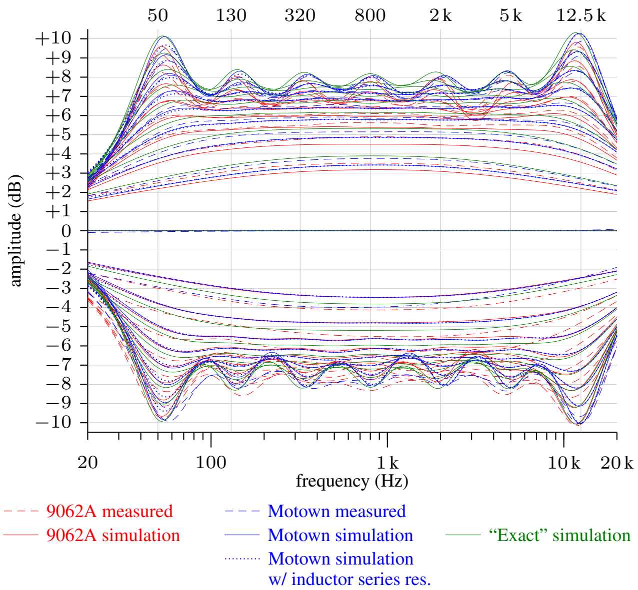



Figure 4: *Magnitude response curves from simultaneously sweeping all seven bands from* $-8$ *to* $+8\,\mathrm{dB}$ *in* $1\,\mathrm{dB}$ *increments. The measured Motown responses for the* $-2$ *to* $-6\,\mathrm{dB}$ *settings are not shown due to a malfunction of the studied unit.*

from a constant-B design (Fig. 5b): (a) The constant-B has an extra variable resistance in parallel with its series RLC tank; (b) $R_\mathrm{h}$ is not present in the constant-B; (c) The constant-B has 4 variable resistors, compared to 2 in the 9062A/Motown; (d) Even ignoring the suppressed values, the two identical horizontal "branches" $R_\mathrm{e} \neq 600\,\Omega$ are different from those of the constant-B ($600\,\Omega$).

Since the constant-*S* ("constant insertion loss" [6]) is described in the Audio Cyclopedia just before the 9062A [8, pp. 311–312], one might wonder whether the constant-B reference is a typo. So, we also show a constant-S EQ in Fig. 5c. Again, neither the circuit topology, absence of $R_\mathrm{h}$, number of variable resistors, nor values of the "branch" resistors match those of the 9062A/Motown (Fig. 5a). Finally, we show a "conventional" Kimball-style [3] bridged-T EQ in Fig. 5d. Again, neither the circuit topology, absence of $R_\mathrm{h}$, nor values of the "branch" resistors match those of the 9062A/Motown (Fig. 5a); however, as with the 9062A/Motown, the number of variable resistors is 2.

So, the core of the 9062A/Motown is neither a constant-B EQ, nor a constant-S EQ, nor a conventional Kimball-style EQ![7]

Various references [1, 2, 9, 17] mention the topology shown in Fig. 5a and seen in the 9062A/Motown, but its importance in variable multi-band EQ design does not appear to have been reported. It goes by several names including "limited-action network" [9], "finite-loss type" (of bridged-T network) [2], and "general form of constant-resistance bridged-T network (a)" [17] or "Bridged-T (Ia)" [1]. For brevity, and to highlight the main difference w.r.t. the classic bridged-T network, we follow Altec's director of engineering, Jim Noble, and call the type of circuit seen in Fig. 5a and the 9062A/Motown a "limited-action network" (or LAN).

[7]The reference to "constant 'K' " [12] may also be misleading, as this normally refers to a specific different filter type. [12] presumably meant it broadly in the sense of "constant-resistance."

## 3. RESISTOR DESIGN

We study the resistors in the LANs. We derive design equations for a single LAN (§3.1), describe how two LANs in cascade creates symmetrical boost and cut (§3.2), describe how variable resistors achieve varying boost (§3.3) and cut (§3.4), comment on a component-saving symmetrical case (§3.5), and derive expressions for ideal and actual amount of required makeup gain (§3.6).

### 3.1. Limited-Action Network

The 9062A/Motown are designed for a $R_0 = 600\,\Omega$ line impedance. The series ($R_\mathrm{e}$) and shunt ($R_\mathrm{h}$) resistors in a LAN (Fig. 6b) depend on a design constant $c$ (that depends on the maximum loss the LAN can provide: $\mathrm{IL}_\mathrm{max}$) and $R_0$. When inserted into a $R_0$ line (e.g., driven by an $R_0$ source and terminated by an $R_0$ load), our single LAN has insertion loss [1, 2, 17]

$$\mathrm{IL}(s) = \frac{2R_0 + (c+1)Z_1(s)}{2R_0 + (c-1)Z_1(s)}\,. \tag{1}$$

When $Z_1(s) = 0$ (a short circuit) and $Z_2(s) = \infty$ (an open circuit), the LAN looks like a no-loss pad (Fig. 7c), the minimum amount of loss the LAN can provide. When $Z_1(s) = \infty$ (an open circuit) and $Z_2(s) = 0$ (a short circuit), the LAN looks like a T-pad attenuator (Fig. 7a) with a certain loss depending on $c$; this is the maximum amount of loss the LAN can provide, and the fact that it is not infinite is the reason for the word "limited" in LAN.[8]

$$\mathrm{IL}_\mathrm{min} = 1 \qquad \longrightarrow 0\ \mathrm{dB} \tag{2}$$

$$\mathrm{IL}_\mathrm{max} = \frac{c+1}{c-1} \qquad \longrightarrow 20\log_{10}\left(\frac{c+1}{c-1}\right)\ \mathrm{dB}\,. \tag{3}$$

The value of $c$ that sets this desired maximum gain is

$$c = \frac{10^{g_\mathrm{max}/20}+1}{10^{g_\mathrm{max}/20}-1}\,. \tag{4}$$

where $g_\mathrm{max} = 20\log_{10}(\mathrm{IL}_\mathrm{max})$. $g_\mathrm{max}$ (in dB) must be positive because a real bridged-T attenuator can only supply positive loss, and is set to the maximum range of gains the LAN will eventually have to provide—either the maximum boost value or the negative of the maximum cut value. This translates to $c \geq 1$, with the case $c = 1$ ($\mathrm{IL}_\mathrm{max} = \infty$) reducing the LAN to a standard BTN with infinite pad loss, by setting $R_\mathrm{e} = R_0$ and $R_\mathrm{h} = 0$.

By consulting the classic equations for the attenuation of a constant-resistance T-pad attenuator [4], we can work out the resistance values that set $\mathrm{IL}_\mathrm{max}$, as also reported in [1, 2, 17]:

$$R_\mathrm{e} = \frac{R_0}{c} \tag{5}$$

$$R_\mathrm{h} = \frac{c^2-1}{2c}R_0\,. \tag{6}$$

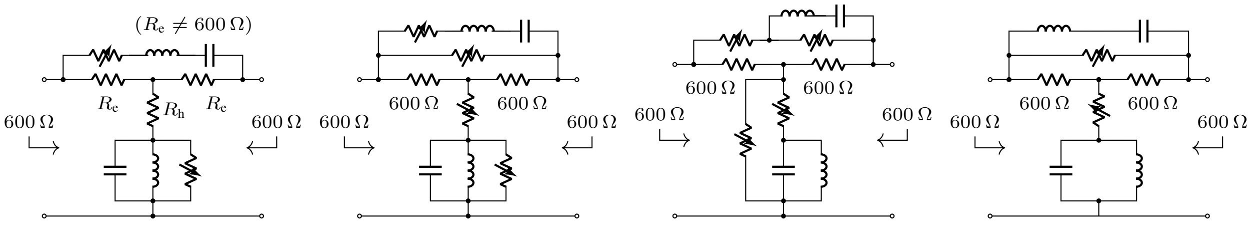
$c \geq 1$ also enforces that $R_\mathrm{e}$ and $R_\mathrm{h}$ are non-negative.

This important feature means that the gain at resonance of any particular frequency band can be set using only simple RLC series or parallel tanks, as opposed to the more elaborate arrangements used in, for instance, constant-B or constant-S EQs.

When $Z_1(s)$ is a parallel RLC tank (and $Z_2(s)$ is the "inverse" series RLC tank), the LAN is in "cut mode" [9]. When $Z_1(s)$ is a series RLC tank (and $Z_2(s)$ is the "inverse" parallel RLC tank),

[8]By contrast, a BTN (Fig. 6a) would have infinite loss (Fig. 7d).

[9]This appears to be the circuit arrangement used in each band of Altec's related "Acousta-Voice" all-cut EQ [18].




(a) *9062A / Motown EQ / LAN.* (b) *Constant-B.* (c) *Constant-S.* (d) *Conventional.*

Figure 5: *Various EQ circuits in boost modes, all designed for* $600\,\Omega$ *line impedance, with most component values suppressed.*

the LAN is in "boost mode"—really "boost with respect to the pad loss." The insertion loss far from the resonant frequency is zero for "cut mode"but equal to the pad loss for "boost mode." Ideally, this discrepancy would be resolved so that moving the gain control knob between positive and negative settings just affects the loss at resonance, but not the loss away from resonance.

### 3.2. Two Cascaded LANS for Symmetrical Boost and Cut

This is solved by using *two* LANs in cascade, as shown in Fig 8f. The left LAN cuts in "cut" mode or passes (does nothing) in "boost" mode, and the right LAN pads in "cut" mode or boosts (again, really "boosts with respect to the pad loss") in "boost" mode. Fig 8a shows the circuit in cut mode and Fig 8d shows it in boost mode. Flat mode (Fig. 8c) can be seen as a special case of either. When $Z_1(s)$ and $Z_2(s)$ are finite and purely resistive, a LAN looks like a Bridged-T attenuator (Fig. 7b). So, varying the resistive parts of $Z_{1\pm}(s)$ and $Z_{2\pm}(s)$ sets a particular boost or cut.

### 3.3. Setting Boost

At their resonant (resp. antiresonant) frequencies, series and parallel RLC tanks look just resistances, meaning a LAN built around one of the pairs looks like a bridged-T pad (Fig. 7b. So, in "boost" mode, at the resonant frequency, our pair of LANs looks like Fig. 8e.

Given $c$ set according to $g_{\max}$, we now study how to set $R_{1+}$ given a desired boost $0 \le g \le g_{\max}$ (in dB). We can solve this by writing out the desired gain with respect to the pad loss.

$$g = 20\log_{10}\left(\frac{2R_0 + (c-1)R_{1+}}{2R_0 + (c+1)R_{1+}} \cdot \frac{c+1}{c-1}\right) \tag{7}$$

Note the fact that the factors inside of the logarithm have been inverted w.r.t. our pad loss equations, since we are thinking of gain now, not insertion loss, and the presence of the factor $\frac{c+1}{c-1}$, which accounts for the fact that we are calculating with respect to the pad loss. This expression can be solved for $R_{1+}$ by

$$R_{1+} = 2\frac{c+1-(c-1)10^{g/20}}{(10^{g/20}-1)(c^2-1)}R_0 \,. \tag{8}$$

### 3.4. Setting Cut

In cut mode, at the resonant frequency, our pair of LANs looks like Fig. 8b, a bridged-T attenuator and a T-pad attenuator in cascade.

Again, given $c$ set according to $g_{\max}$, we now study how to set $R_{1-}$ given a desired cut $-g_{\max} \le g \le 0$. We can solve this by writing out the desired gain with respect to the pad loss.

$$g = 20\log_{10}\left(\frac{2R_0 + (c-1)R_{1-}}{2R_0 + (c+1)R_{1-}}\right) \,. \tag{9}$$

This expression can be solved for $R_{1-}$ by

$$R_{1-} = 2\frac{1-10^{g/20}}{(c+1)10^{g/20}-(c-1)}R_0 \,. \tag{10}$$

### 3.5. Reusing resistors

There is one special value of $c$, which we will call $c_\star$, that also allows the same set of impedances to be used in the boost and cut cases. To get this, we need to have

$$R_{1+} = R_{2-} = \frac{R_0^2}{R_{1-}} = \frac{R_0^2}{R_{2+}} \,. \tag{11}$$

Plugging this into the gain equations gives

$$\left(\frac{2R_0 + (c_\star+1)R_{1+}}{2R_0 + (c_\star-1)R_{1+}}\right)\left(\frac{2R_{1+} + (c_\star+1)R_0}{2R_{1+} + (c_\star-1)R_0}\right) = \frac{c_\star+1}{c_\star-1} \tag{12}$$

This can be solved for $c_\star$ by

$$c_\star = \pm\sqrt{5} \approx \pm 2.2361 \,. \tag{13}$$

Because we must have $c > 1$ for passivity, we can discard the negative solution and only consider $c_\star = \sqrt{5}$. The associated insertion loss has a relationship to the "golden ratio" $\varphi = \frac{1+\sqrt{5}}{2}$. In fact, $\frac{c_\star+1}{c_\star-1} = \varphi^2 = \varphi + 1$. $c_\star$ is associated with a pad loss and hence a maximum boost/cut of

$$g_\star = 20\log_{10}\left(\frac{\sqrt{5}+1}{\sqrt{5}-1}\right) \approx 8.3595 \text{ dB} \,. \tag{14}$$

Interestingly, this is the same condition (given as 8.36 dB) on symmetry that constant-B attenuators have, as mentioned, but not derived, in the Audio Cyclopedia [8, p. 311].

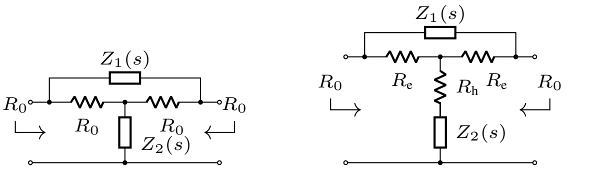
Given the 16 dB flat insertion loss for two stages, it is apparent that the designers of the 9062A/Motown designed for a pad loss of 8 dB, which would give a $c$ value of

$$c_8 = \frac{10^{8/20}+1}{10^{8/20}-1} \approx 2.3229 \,. \tag{15}$$

Tab. 2 shows the $c$, $\mathrm{IL}_{\max}$, $R_e$, $R_h$, $R_{1\pm}$, $R_{2\pm}$ measured from the 9062A/Motown, and "Exact" quantities that come from running the proposed design procedure with $c = c_\star$.

### 3.6. Makeup gain

Because of the pad loss of each of the two "pad" LANs, at the "flat" setting of this style of EQ, there will not be zero insertion loss, but rather twice the amount of pad loss of one LAN. On top




(a) *Bridged-T Network (BTN).* (b) *Limited-Action Network (LAN).*

Figure 6: *Bridged-T Network and Limited-Action Network.*


(a) *T-Pad Attenuator.* (b) *Bridged-T Attenuator.*


(c) *No Padding.* (d) *Total Pad.*

Figure 7: *Several resistive constant-resistance networks.*

of this, thinking of the actual magnitude response if it is driven by an $R_{\text{source}} = R_0$ resistive voltage source and terminated by an $R_{\text{load}} = R_0$ load resistor, there will be an additional insertion loss of $-20\log_{10}(1/2) \approx 6.0206\,\text{dB}$. So all together, to get the "flat" setting to actually sit at 0 dB, we need a makeup gain (in dB) of

$$-20\log_{10}\left(\frac{1}{2}\left(\frac{c-1}{c+1}\right)^2\right) . \tag{16}$$

Using $c = c_\star$, this is ≈22.7396 dB. If we were dealing with the exact resistor values observed or measured from the real device, rather than idealizations, we may not be able to rely on all the component values perfectly satisfying all of the design equations. Still assuming that $R_{\text{source}} = R_0$ and $R_{\text{load}} = R_0$, and assuming that all of the LANs have the same $R_{\text{e}}$ and $R_{\text{h}}$, the precise value of makeup gain in terms of the component values can be derived using modified nodal analysis [19] as

$$\frac{2(R_0^2(R_{\text{e}}+R_{\text{h}}) + (2R_0+3R_{\text{h}})R_{\text{e}}^2 + 4R_0R_{\text{e}}R_{\text{h}} + R_{\text{e}}^3 + (2R_{\text{e}}+R_0)R_{\text{h}}^2)}{R_0R_{\text{h}}^2} . \tag{17}$$

For the actual component values we found in the 9062A, this would be ≈21.5836 dB of makeup gain. In practice, this would have needed to be supplied by an external amplifier in the original 9062A. In the Motown, a built-in amplifier supplies this gain.

## 4. REACTANCE DESIGN

For a center frequency (in Hz) $f_{\text{c}}$, bandwidth $B$, given $c$, and $R_{1+}$ that sets a certain boost, $20\log_{10}\text{IL}(f_{\text{c}}B) = \frac{1}{2}20\log_{10}\text{IL}(f_{\text{c}})$ is laboriously solved by

$$C_{1+} = \frac{B^2-1}{2\pi f_{\text{c}}B}\sqrt{\frac{c^2-1}{4R_0(R_0+R_{1+}c)+R_{1+}^2(c^2-1)}} \tag{18}$$

$$L_{1+} = \frac{B}{(B^2-1)2\pi f_{\text{c}}}\sqrt{\frac{4R_0(R_0+R_{1+}c)+R_{1+}^2(c^2-1)}{c^2-1}} . \tag{19}$$

Here and throughout, due to the inverse impedance principle, we have $C_{2\pm} = L_{1\pm}/R_0^2$ and $L_{2\pm} = C_{1\pm}R_0^2$.

For cut mode, $20\log_{10}\text{IL}(f_{\text{c}}B) = \frac{1}{2}20\log_{10}\text{IL}(f_{\text{c}})$ is laboriously solved by

$$C_{1-} = \frac{B}{(B^2-1)2\pi f_{\text{c}}}\frac{\sqrt{4R_0(R_0+R_{1-}c)+R_{1-}^2(c^2-1)}}{2R_{1-}R_0} \tag{20}$$

$$L_{1-} = \frac{B^2-1}{2\pi f_{\text{c}}B}\frac{2R_{1-}R_0}{\sqrt{4R_0(R_0+R_{1-}c)+R_{1-}^2(c^2-1)}} . \tag{21}$$

Variable inductors and capacitors are expensive, so in constant-resistance EQs (e.g. [6]), commonly a single set of reactances is used per boost and cut mode, designed for the maximum boost or cut gain, and then reused for all other gains, causing proportional-bandwidth behavior (recall Fig. 3b), but preserving the correct peak amplitude and frequency. As with the resistors, if the LAN is designed with $c = c_\star$, the same set may be used in boost and cut mode. Using these two "tricks" together allows a circuit to use many fewer reactances per band, as low as 2 each of capacitors and inductors, compared with 32 when neither "trick" was not used.

Inductors have a finite series resistance that can be important in audio circuits, especially for large inductances, usually corresponding to low frequency resonances. In the Motown, certain inductors have a labelled series inductance. In Figs. 2 and 4, it can been seen that including this series resistance in the simulations improves the match of the simulation's magnitude response to the real device's measured magnitude response.

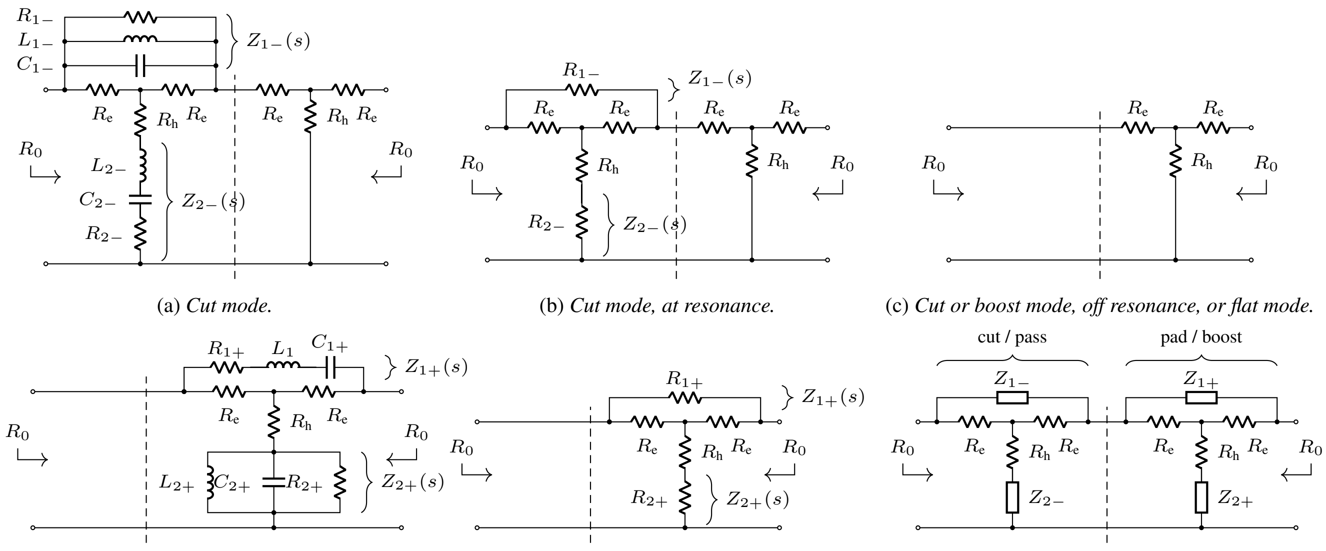
## 5. MULTIBAND DESIGN

The arrangement used in the 9062A/Motown, including appropriate resistive source and load, is shown in Fig. 9. Each generic impedance is chosen among shorts, open circuits, and series or parallel RLC tanks according to Tab. 1.

Let us return to the mystery of how the 9062A/Motown have only ≈16 dB is flat insertion loss, when they have 7 bands, each of which we would expect to use an entire pair of LANs, each contributing 8 dB for a total of 56 dB. The designers came up with an ingenious idea: bundling multiple frequency bands into a single pair of LANs. They grouped all the odd-numbered bands ($n = \{1, 3, 5, 7\}$ or 50, 320, 2000, and 12,500 Hz bands) into one LAN pair and all the even-numbered bands ($n = \{2, 4, 6\}$ or 120, 800, 5000 Hz bands) into a second. This allowed them to save 40 dB of insertion loss, so that 40 dB less makeup gain is required to use the unit, and noise levels are 40 dB lower.

They were able to do this because, "sufficiently far" from the center frequency of each band, its $Z_{1-}$ and $Z_{2+}$ look like short circuits, allowing them to be put in *series* with any other impedance without affecting it, and its $Z_{1+}$ and $Z_{2-}$ look like open circuits, allowing them to be put in *parallel* with any other impedance without affecting it. This causes acceptably small interactions within groups.[10] $Z_1$ and $Z_2$ in constant-B, constant-S, and conventional EQs do not have this property, disqualifying multiple bands from being used within a single block.

[10] In theory, all 7 bands could have been bundled into a single LAN pair, getting the insertion loss down to an incredible 8 dB. However, this would cause excessive and deleterious band interactions.




(a) *Cut mode.* (b) *Cut mode, at resonance.* (c) *Cut or boost mode, off resonance, or flat mode.*

(d) *Boost mode.* (e) *Boost mode, at resonance.* (f) *A combined multiband cut/flat/boost circuit.*

Figure 8: *A single frequency band in cut, flat, and boost mode, on and off resonance, and the general combined LAN pair.*

## 6. WAVE DIGITAL FILTER MODEL

We implemented digital models of the 9062A/Motown using Wave Digital Filters (WDFs) [20–24]. There is not room to give a full accounting of its structure, but it follows immediately from the topology of the circuit [21]. Several bridged-T topologies arise and can be handled using known approaches [22–24]. The WDF model is augmented with the appropriate amount of makeup gain.

All WDFs were run at a sampling rate of $44{,}100\,\text{Hz}$, with $\times 8$ oversampling so that the reactances can be discretized with the standard, non-warped, bilinear transform. We also tried using our design equations to pre-warp the band frequencies and amplitudes individually, essentially designing a new circuit that precompensates for the frequency depressing and bandwidth shrinking properties of the bilinear transform [25, 26] and allow a lower oversampling rate to be used in practice, similar to using a different discretization scheme for each reactance [27]

Magnitude responses for single bands at $\pm 8\,\text{dB}$ (Fig. 2), sweeping the gain of the $800\,\text{Hz}$ band's gain (Fig. 3a), and sweeping all bands together (Fig. 4), as well the model's relationship between bandwidth and gain (Fig. 3b) are given. For all of these, we simulate a model using the component values for the 9062A, the Motown, and an "Exact" version where the resistor and capacitor values are designed using the design equations derived in this paper. Across the board, we see broad agreement between the measured, simulated, and datasheet responses, as well as broad agreement between the 9062A, Motown, and "Exact" varieties.

## 7. CONCLUSION

We used published info, electrical measurements, board traces, and mathematical derivations to study two classic 7-band, passive, constant-resistance EQs: the Altec 9062A and the Motown EQ. We showed that they use a special bridged-T network variant called a "Limited Action Network" (LAN) [9], which enabled the designers to bundle several frequency bands together, achieving a lower part count and a lower flat insertion loss than a "textbook" design. Despite claims of otherwise reputable sources, we showed that these EQs are *NOT based on constant-B EQs*. We derived design principles and equations for all the component values.

Comparing the resistor values (Tab. 2) from the 9062A/Motown and our proposed design procedures—Although the frequency responses are very similar, and the resistor values in the 9062A and Motown closely match one another, they do not closely match our "Exact" resistors, and conspicuously only approximately satisfy "inverse resistance" principle. We thought this could be a consequence of approximating $g_{\text{max}} = g_{\star}$ as $g_{\text{max}} = 8$, but investigated obvious ways to compromise between the no-longer-perfectly symmetrical resistor values, including averaging, geometric mean, min, and max, and none of these yielded values very close to those the 9062A/Motown. So, it seems unlikely that their values came from "compromises" of the presented procedure. They may have adjusted the resistances to compensate in some way for known series resistances in the inductors; however even in the lower bands where these resistances are substantial, they are typically much smaller than the difference between the "exact" and tabulated resistances. Perhaps the original designers did not even derive the full resistor-design procedure, or that they used some approximation. We cannot say anything conclusive.

Beyond setting the historical record straight and explaining a hitherto misunderstood family of important audio circuit designs, our findings also enable extending the original 9062A/Motown designs. We derived a way to have shelf-style bands, as in [6], which are useful alternatives for the lowest and highest bands. We can add extra bands, e.g. another one below $50\,\text{Hz}$ and/or above $12.5\,\text{kHz}$, or, e.g., create alternate versions with many more 1/3-octave spaced frequency bands. We can control gains continuously, rather than just at integer detents, and extend the gain range beyond $\pm 8\,\text{dB}$. Fig. 3b shows the bandwidth massively increases between $-1$ and $+1\,\text{dB}$, which would damage band independence; similarly, with increased gain range, bandwidth can narrow significantly outside of the $[-8, +8]\text{dB}$ range. We can control

| gain | $Z_{1-,n}$ | $Z_{2-,n}$ | $Z_{1+,n}$ | $Z_{2+,n}$ |
|---|---|---|---|---|
| boost ($g_n > 0$) | | | $R_{1+,n}$ $L_{1+,n}$ $C_{1+,n}$ | $R_{2+,n}$ $L_{2+,n}$ $C_{2+,n}$ |
| flat ($g_n = 0$) | | | | |
| cut ($g_n < 0$) | $R_{1-,n}$ $L_{1-,n}$ $C_{1-,n}$ | $R_{2-,n}$ $L_{2-,n}$ $C_{2-,n}$ | | |

Table 1: *The identity (short or open circuit, series or parallel RLC tank) of the four impedances in each band $n \in 1 \cdots 7$.*

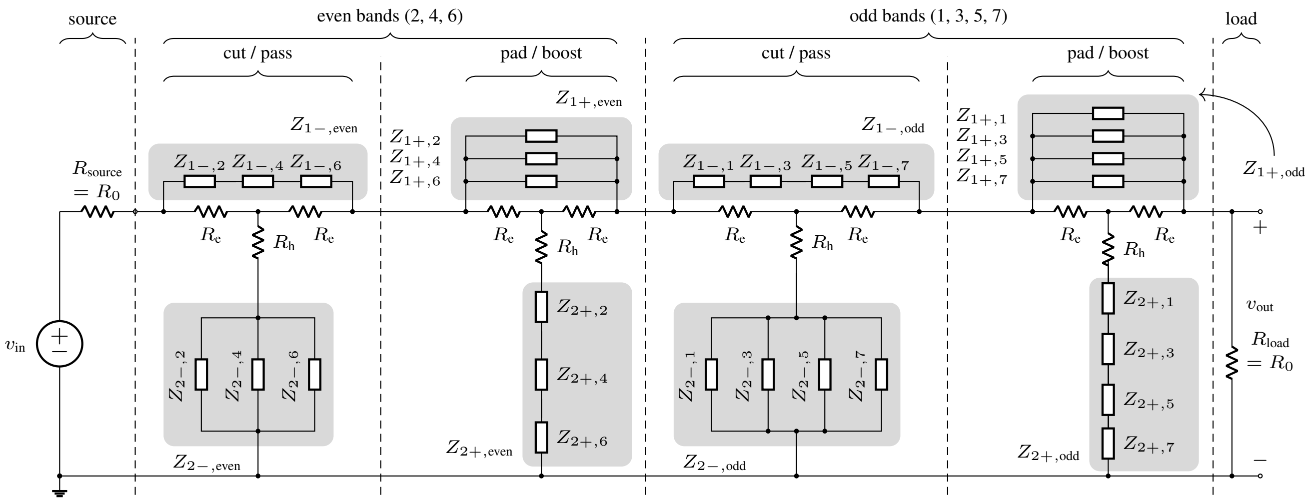



Figure 9: *The full seven-band EQ circuit used in the Altec 9062A and the Motown EQ.*

the relationship between gain and bandwidth arbitrarily, since in a digital model we can vary reactances freely. This enables creating true constant-bandwidth behavior (or any compromise between proportional- and constant-bandwidth), and “clamping” the bandwidth retains the original behavior in the original gain range, but doesn't allow higher or lower bandwidth—Fig. 3b shows an example of this. Despite the practical reasons to bundle all bands into two LAN pairs in the original design, we created alternative architectures where each band has its own LAN, eliminating all band interaction, or other groupings: for instance 3 interleaved groups of 3, 2, and 2 bands. In a digital model, the makeup gain required to correct for the flat insertion loss comes with none of the chilling downsides of the original analog implementation.

Unfortunately, there is not room in this short conference paper to demonstrate these new features and alternative architectures.

In many circuit designs based on impedance matching, unique and useful musical effects can be created by altering the input and output impedances [11]. The author has also had success using this technique on models of the Altec 9069B and the Allison Labs 2AB. Informal experiments along these lines were done with our 9062A/Motown model, however they did not seem to yield many useful or musical results. This may be a design feature of LANs—An Altec service note [9] says that, for LANs, “Response curves are virtually unaffected by source and load impedances.” Although this could be another advantage of the LAN, it unfortunately represents a bit of an aesthetic dead-end for would-be circuit-benders!

## 8. ACKNOWLEDGMENTS

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| | Altec 9062A | | | | Motown EQ | | | | Exact Design | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| dB ideal gain | Ω $R_{1+}$ $R_{2-}$ | Ω $R_{2+}$ $R_{1-}$ | dB + | dB − | Ω $R_{1+}$ $R_{2-}$ | Ω $R_{2+}$ $R_{1-}$ | dB + | dB − | Ω $R_{1+}$ $R_{2-}$ | Ω $R_{2+}$ $R_{1-}$ | dB + | dB − |
| $\pm 8$ | $\infty$ | 0 | $+7.5203$ | $-7.5824$ | $\infty$ | 0 | $+7.8547$ | $-7.8667$ | 13827.6 | 26.0 | $+8$ | $-8$ |
| $\pm 7$ | 4381 | 56 | $+6.6992$ | $-6.6631$ | 4543.3 | 57.6 | $+7.0311$ | $-7.0918$ | 3170.4 | 113.6 | $+7$ | $-7$ |
| $\pm 6$ | 1381 | 103 | $+5.8873$ | $-5.9180$ | 1383.3 | 104.2 | $+6.0728$ | $-6.1571$ | 1551.5 | 232.0 | $+6$ | $-6$ |
| $\pm 5$ | 761 | 213 | $+4.9299$ | $-4.9772$ | 749.3 | 211.2 | $+5.0778$ | $-5.1769$ | 899.8 | 400.1 | $+5$ | $-5$ |
| $\pm 4$ | 461 | 373 | $+3.9953$ | $-4.0655$ | 455.3 | 376.2 | $+4.1062$ | $-4.2120$ | 549.6 | 655.0 | $+4$ | $-4$ |
| $\pm 3$ | 241 | 593 | $+3.0569$ | $-3.0311$ | 234.3 | 608.2 | $+3.0662$ | $-3.1504$ | 332.2 | 1083.6 | $+3$ | $-3$ |
| $\pm 2$ | 159 | 1273 | $+1.9972$ | $-2.1234$ | 153.7 | 1306.2 | $+2.0434$ | $-2.1410$ | 185.0 | 1946.4 | $+2$ | $-2$ |
| $\pm 1$ | 91 | 4273 | $+0.9281$ | $-1.1408$ | 85.6 | 4316.2 | $+0.9928$ | $-1.0452$ | 79.2 | 4546.5 | $+1$ | $-1$ |
| 0 | 0 | $\infty$ | 0 | 0 | 0 | $\infty$ | 0 | 0 | 0 | $\infty$ | 0 | 0 |
| $R_e$ | $240\,\Omega$ | | | | $249\,\Omega$ | | | | $268.3\,\Omega$ | | | |
| $R_h$ | $560\,\Omega$ | | | | $536\,\Omega$ | | | | $536.7\,\Omega$ | | | |
| $c$ | $\approx 2.3229$ | | | | $\approx 2.3229$ | | | | $c_\star = \sqrt{5} \approx 2.2361$ | | | |
| $g_{\max}$ | $\approx 8\,\mathrm{dB}$ | | | | $\approx 8\,\mathrm{dB}$ | | | | $g_\star \approx 8.3595\,\mathrm{dB}$ | | | |

Table 2: *Tabulated Altec 9062A and Motown EQ resistor values and gains, compared with "Exact" component values derived from the design equations (assuming $R_0 = 600\,\Omega$) and ideal boost (+) and cut (−) gains. These resistors are identical for all seven bands.*

| | | Altec 9062A | | | | Motown EQ | | | | Exact Design | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Hz band | $n$ | µF $C_{1+,n}$ $C_{2-,n}$ | mH $L_{1+,n}$ $L_{2-,n}$ | µF $C_{2+,n}$ $C_{1-,n}$ | mH $L_{2+,n}$ $L_{1-,n}$ | µF $C_{1+,n}$ $C_{2-,n}$ | mH $L_{1+,n}$ $L_{2-,n}$ | µF $C_{2+,n}$ $C_{1-,n}$ | mH $L_{2+,n}$ $L_{1-,n}$ | µF $C_{1+,n}$ $C_{2-,n}$ | mH $L_{1+,n}$ $L_{2-,n}$ | µF $C_{2+,n}$ $C_{1-,n}$ | mH $L_{2+,n}$ $L_{1-,n}$ |
| 50 | 1 | 5.66 | 1790 | 4.97 | 2037 | 5.472 | 1730 | 4.806 | 1970 | 5.492 | 1845 | 5.125 | 1977 |
| 130 | 2 | 2.18 | 689 | 1.91 | 783 | 2.125 | 649.5 | 1.804 | 765 | 2.112 | 709.6 | 1.971 | 760.4 |
| 320 | 3 | 0.884 | 280 | 0.77 | 318 | 0.850 | 263.1 | 0.7307 | 306.1 | 0.8581 | 288.3 | 0.8008 | 308.9 |
| 800 | 4 | 0.354 | 112 | 0.311 | 127 | 0.339 | 106.5 | 0.2960 | 122 | 0.3432 | 115.3 | 0.3203 | 123.6 |
| 2000 | 5 | 0.141 | 44.8 | 0.124 | 50.9 | 0.136 | 40.3 | 0.1119 | 48.9 | 0.1373 | 46.13 | 0.1281 | 49.42 |
| 5000 | 6 | 0.0566 | 17.685 | 0.0497 | 17.685 | 0.055 | 17.0 | 0.04723 | 19.7 | 0.05492 | 18.45 | 0.05125 | 19.77 |
| 12500 | 7 | 0.0226 | 7.16 | 0.0199 | 8.14 | 0.022 | 6.8 | 0.01889 | 7.81 | 0.02197 | 7.380 | 0.02050 | 7.908 |

Table 3: *The tabulated reactance values from the Altec 9062A and Motown EQ, compared with the "exact" component values derived from the design equations in the paper. For these "exact" designs, the center frequencies on the front panels are used, the extracted bandwidth ($B = 1.6803$, or $0.7487$ octaves) for the Altec service notes, a pad loss of $g_\star \approx 8.3595\,\mathrm{dB}$, and a reactance design gain of $8\,\mathrm{dB}$ are used. The shaded Motown values were missing from the original device or inaccessible, so have been approximately reconstructed using on appropriate interpolations of information from other bands, design equations, and measurements.*


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