

MEASURING SENSORY CONSONANCE BY AUDITORY MODELLING

Esben Skovenborg

Søren H. Nielsen

Dept. of Computer Science, University of Aarhus
Åbogade 34, DK-8200 Aarhus N, Denmark
esben@skovenborg.dk

TC Electronic A/S
Sindalsvej 34, DK-8240 Risskov, Denmark
soerenn@tcelectronic.com

ABSTRACT

A current model of pitch perception is based on cochlear filtering followed by a periodicity detection. Such a computational model is implemented and then extended to characterise the sensory consonance of pitch intervals. A simple scalar measure of sensory consonance is developed, and to evaluate this perceptually related feature extraction the consonance is computed for musical intervals. The relation of consonance and dissonance to the psychoacoustic notions of roughness and critical bandwidth is discussed.

1. INTRODUCTION

When listening to tonal music, some pitches are perceived individually whereas others fuse together to form structures such as chords. A single pitch can be characterised by a frequency, or in a musical context by a scale degree; but when two tones are heard together the sensation depends primarily on the *interval* between the pitches. Such pitch intervals are commonly characterised by their *consonance* – a musical concept which also has roots in psychoacoustics.

From an auditory modelling point of view, it would therefore be interesting if a model could be constructed that was able to deal with pitch as well as consonance. This idea is explored in the work documented here, based on an earlier project [1]. An established auditory model of pitch is implemented. It is then modified into a simple model of consonance perception. The output of this computational model is then studied for input consisting of pitch intervals with varying expected consonance, particularly the intervals found in conventional musical scales. Based on this study, the model is evaluated against results of previous psychoacoustic investigations into consonance.

The sensory consonance is related to certain audio descriptors in MPEG-7 [2][3]. As the present consonance measure is based on a psychoacoustic model, instead of simple time- or frequency domain analysis, it can be considered as a *high-level feature*. Some of the attributes described in MPEG-7 concern only monophonic sounds whereas consonance takes into account polyphonic effects.

One of the possible applications of the described model of sensory consonance could be within adaptive sound processing. Along with other attributes such as loudness and pitch, the consonance could control one or several parameters in a sound (or video) processing algorithm. Some examples of adaptive sound processing have recently been described in the DAFX community [4][5][6].

2. A MODEL OF PITCH PERCEPTION

As a starting point for the model of consonance, a recent model of pitch perception was used. This particular type of model has the feature of being able to predict the pitch frequency for a range of different types of pitched signals [8]. Basically, the pitch is extracted from a periodicity measurement on the responses of each auditory channel (the *correlogram*). The construction and evaluation of this type of perceptual pitch model has been the topic of several projects [9][10][11].

The list below outlines the computational pitch model used in this work. Steps 1-6 were implemented using modules from the HUTear MATLAB toolbox [12], and the later steps were developed from scratch, in the context of the project [1]. A sampling frequency of 44.1 kHz is used throughout the model.

- 1) **Pre-processing of audio stimulus:** Scale the level of the input to the desired SPL; 60 dB is used here.
- 2) **Simulate the frequency response of the outer ear:** Free sound field response (MAF).
- 3) **Simulate the frequency response of the middle ear:** By considering the outer and middle ear as a linear time-invariant system, the combined frequency response is modelled using a fixed FIR filter.
- 4) **Simulate the frequency analysis of the cochlea:** A bank of 64 gammatone filters; centre-frequencies are equidistant on the Bark scale, corresponding to a resolution of 1/2 Bark.
- 5) **Simple inner hair cell model:** Half-wave rectification, simulating the 'phase-locking' of the mechanical to neural transduction.
- 6) Low-pass filtering, 1kHz, simulating 'neural bandwidth saturation'.
- 7) **Within-channel periodicity detection; correlogram:** Calculate the autocorrelation, using FFT, for each channel.
- 8) Summation of all autocorrelations across channels, to produce the SummaryAutoCorrelation curve (SAC).
- 9) **Find dominant period:** Locate the first maximum after the 0th lag, and improve estimate of peak by interpolation.
- 10) **Estimate 'best' pitch frequency:** Convert the location of the SAC maximum into a frequency of periodicity, and use it as pitch estimate.

Each of the steps 1-6 of the pitch model are simple approximations of the corresponding auditory functionality (see e.g. [13]) – for instance, steps 5-6 model only the average behaviour of numerous inner hair cells and auditory nerve fibres.

The model is entirely *data-driven* in the sense that there is no feedback to any lower layers, and also no adaptation to the specific stimuli takes place. Moreover, the model is monaural and most temporal auditory aspects are ignored. As all the signals analysed in this work are stationary and in most cases periodic, issues of time/frequency-resolution, windowing, etc. are also ignored.

2.1. Evaluation of the Pitch Model

The implemented pitch model has been tested using 4 different kinds of monophonic static pitch stimuli; the frequency of each pitch was in the range 220-440 Hz.

- **pure tones:** synthesized sinusoidal waveforms
- **harmonic tones:** consist of a full harmonic overtone series, i.e. all integer-multiples of the fundamental, below the Nyquist frequency; the amplitude of each harmonic partial is scaled corresponding to an attenuation of 6 dB/octave
- **virtual pitch:** harmonic tones, with missing fundamental and lowest harmonics
- **comb-filtered noise:** from track 51 of the CD [14]; an example of *atonal pitch*, i.e. a tone with a continuous spectrum

In each of the above cases the computational model of pitch produced the perceptually correct estimate of the pitch frequency [1]. It therefore seems appropriate to employ as basis for the consonance model (section 4).

3. CONSONANCE OF TONES

When two tones are presented together the resulting sensation can be qualitatively different from that of a single tone. Two simultaneous tones may *fuse* together, in which case the *interval* (the frequency ratio) between the two tones is highly significant to the perceived sound – a feature that is heavily used in music.

The concept of **musical consonance** can be accounted for by two separate classes of phenomena [15]: one is called *sensory consonance* and is based on the psychoacoustically well-defined concepts of roughness, sharpness (kind of a spectral-envelope weighted loudness), and tonalness (the opposite of noisiness). These three qualities apply to any type of sound. The other contributing factors to musical consonance, jointly denoted *harmony*, are specific to musical sound and concern certain notions from music theory [17].

The model of consonance developed here is based on an auditory model, and we do not wish to impose any presumptions about how various types of musical stimuli are perceived – instead we shall study the output of the model to see if any musical interpretations are possible. Therefore, **the aspect of musical consonance that we endeavour to model is the sensory consonance.**

3.1. Musical Scales

Music which contains pitched tones is generally based on *scales*. A scale is a set of pitches with a fixed frequency relationship. In western music, the octave is divided into 12 *semitones* (the chromatic scale) on which all other scales are built [18].

The tempered (or *equal tempered* or *well-tempered*) scale was invented to be a practical alternative to the *just intonation*. Each octave is divided equally into 12 semitones, so that the frequency ratio between any two neighbouring tones is $\sqrt[12]{2}$. This implies that the tempered scale is *symmetrical* and the same tuning can hence be used for playing in all keys [19], as exploited by J.S. Bach in his composition 'Das wohl-temperierte Clavier' (1722). However, the tempered scale is a compromise because some of its intervals are relatively far from the corresponding 'pure' intervals of just scale, in which the fundamental frequencies are *in the ratio of small integer numbers*, e.g. the interval of a pure perfect fifth with the frequency ratio of 3/2.

Today, 'western' music is generally played using the tempered scale and its intervals are therefore of special interest. Particularly fixed-scale instruments (such as the piano) employ a tuning similar to the equal tempered.

3.2. Consonance and Roughness

Von Helmholtz discovered that dissonance occurs when partials of two tones produce amplitude fluctuations (beating) in a certain frequency range. The more partials of one tone that coincide with the partials of the other, the less chance that beating in this range will contribute to dissonance [19][20]. Consonance would then be the absence of such beating partials within critical bands. Thus, we shall henceforth assume that dissonance is simply the *opposite* of consonance.

Dissonance is related to the sensation *roughness*, identified in psychoacoustics. Roughness can be induced by an amplitude-modulated sine tone, and is strongest when the tone is 100% modulated at a modulation frequency of 70 Hz; but roughness is perceived with modulation frequencies from 15-250 Hz when the carrier is at 1 kHz [15]. Both critical bandwidth and the limited temporal resolution of the auditory system contribute to defining this roughness range. The perceived roughness furthermore depends on the loudness of the stimulus. A pair of pure tones will cause an amplitude fluctuation with a frequency that is the difference between the frequencies of the tones. This situation could also induce roughness, if the tones were partials, isolated within a critical bandwidth.

To summarise, **consonant intervals of harmonic tones have fewer harmonics with frequency differences within the roughness range.** But these (possibly unresolved) **harmonics are also affected by the spectrum of the tones;** i.e. the number of harmonics and their strength (contributing to the perceived timbre). The roughness is strongest when the interacting harmonics are both strong and equally strong.

An *auditory spectrogram* (or *cochleagram*) can be constructed by plotting the output from step 6 in the implemented pitch model, displaying the average auditory nerve firing for each auditory channel over time. Two specific intervals were chosen because they are renowned for being especially consonant and dissonant, respectively [21], though consisting of pitches close in frequency: the interval of a perfect fifth and of a tritone (the interval equal to the sum of three whole tones). Each individual tone was synthesized like the *harmonic tones* described in section 2.1. Figure 1 and Figure 2 contain the auditory spectrograms for these two contrasting intervals, and the connection to roughness is clearly visible: The regularity of the

fifth, caused by its 3/2 ratio, did not produce the low-frequency amplitude-modulation-like behaviour evident in some of the channels for the tritone, noticeably around 1.5 and 3 kHz.

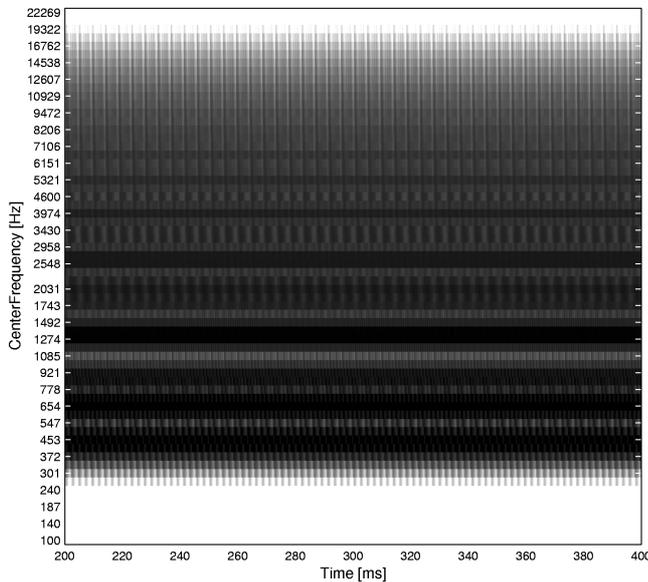


Figure 1: Auditory spectrogram of the interval of a perfect fifth (just intonation), using harmonic tones. Pitch 1 is 440 Hz, pitch 2 is 660 Hz; the interval is 702 cent. The range from weak to strong (white to black) is around 50 dB.

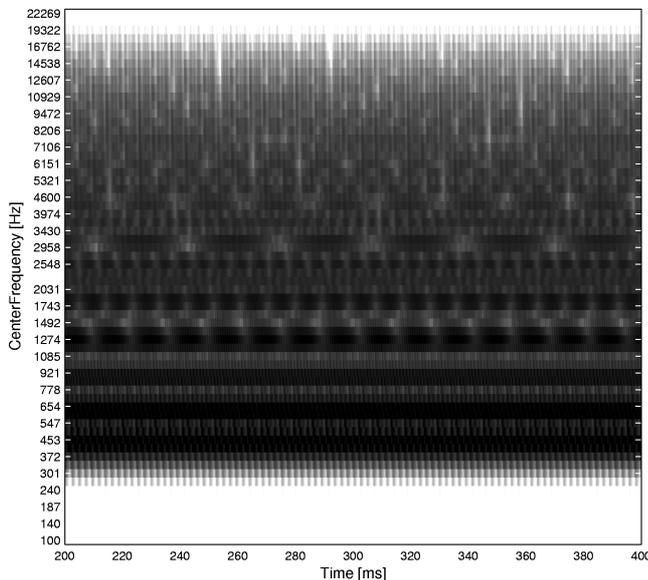


Figure 2: Auditory spectrogram of the interval of a tritone (equal tempered), using harmonic tones. Pitch 1 is 440 Hz, pitch 2 is 622 Hz; the interval is 600 cent. The same intensity scale as in Figure 1 is used.

4. A MODEL OF CONSONANCE

In order to construct a model of the sensory consonance of pitch intervals, the pitch model presented in section 2 needs to be modified. The consonance model needs to capture the varying roughness induced by tone intervals. This is implemented by changing the last four steps of the pitch model to the following (steps 1-6 are identical to those in the pitch model):

- 7) **Within-channel spectral analysis:** Calculate the power spectral density (PSD), using FFT, for each channel.
- 8) Summation of PSDs across all channels with centre-frequencies above ~ 3 Bark.
- 9) **Find the power related to roughness:** Calculate the mean value of the SummaryPSD in the frequency range approx. 15 - 300 Hz.
- 10) **Estimate consonance measure:** Convert the mean into a scalar measure of sensory consonance: $SC = 100 - \text{mean}$.

Note that the pitch and consonance models have resemblance, in addition to sharing the first 6 steps: The autocorrelation and the spectral density estimation (steps 7) are similar operations, and the subsequent summations (steps 8) are equivalent.

The output of the consonance model is a scalar parameter, SC , which we shall denote the **sensory consonance measure** corresponding to the input (stimulus) fed to the model at step 1.

The sensory consonance measure is thus defined as:

$$SC = 100 - \left(\frac{1}{\#f} \sum_{f=f_{Rlo}}^{f_{Rhi}} \left(\frac{1}{\#z} \sum_{z=z_{lo}}^{z_{max}} PSD(z, f) \right) \right) \quad (1)$$

where –

- PSD is the power spectral density, with values in dB; z is a channel in the gammatone filterbank, and f is a frequency band in the FFT used to calculate the PSD.
- f_{Rlo} should be around 15 Hz, and f_{Rhi} around 250-300 Hz, to capture the 'roughness spectrum' (section 3.2)
- $\#f$ is the number of frequency-bands in the PSD between f_{Rlo} and f_{Rhi}
- z_{lo} should be a filterbank channel below the lowest channel containing unresolved partials. (This constraint is dependent on the stimulus, but a simple solution is to use a channel below the fundamental frequency of the lowest pitch occurring in the stimulus.)
- z_{max} is the highest auditory channel in the cochlea model
- the lower cut-off frequency of the channel z_{lo} should be above f_{Rhi}
- $\#z$ is the number of channels between z_{lo} and z_{max} (1/2 Bark resolution is used here)

In Equation 1, the summation finds the total power in the cochleagram within the roughness range, caused by beats of adjacent unresolved partials. The point of the $100 - (\cdot)$ is simply to make a scale with $SC \approx 0$ for very dissonant intervals, and $SC \approx 100$ for very consonant intervals (SC is *not* in percent). The definition of SC is based on experiments with various tone intervals, and thus the range of SC is not theoretically bounded by 0 and 100.

Note that the *SC* measure does not (yet) take into account the dependency of roughness on frequency region, and using an unweighted average in the integration is also a simplification.

Comparing the present model of sensory consonance to the model developed by Aures [22][23], there are some differences. Aures models the sensory consonance as a scalar formed as the product of four psychoacoustic quantities transformed and scaled appropriately: Roughness, sharpness, tonalness and loudness. The four psychoacoustic quantities were calculated separately using well-known and partly adapted models. As reference, listening experiments on signals exhibiting varying degrees of the four quantities, were made. The two dominating quantities in the Aures model are roughness and tonalness. It remains to be investigated whether the four quantities could be reduced to fewer using formal multidimensional scaling techniques.

4.1. Evaluation of the Consonance Model

To evaluate the consonance model, the measured sensory consonance of intervals was compared with established results from psychoacoustic experiments with subjective judgment of consonance/dissonance.

Plomp and Levelt conducted a set of psychoacoustic experiments in which the subjects rated to what degree pairs of pure tones sounded consonant or dissonant [24]. The frequency difference of the two tones would vary around a fixed mean frequency. Figure 4 shows one of the resulting consonance-rating curves, for 14 intervals in the range 9-900 Hz, each with a geometric mean frequency of 500 Hz. The curve is relatively smooth and without any peaks at the intervals presumed consonant. Seemingly, the consonance ratings depended on the *distance* rather than the ratio between the tones' frequencies. However, it was realised that the shape of the curves, for various mean frequencies, could all be explained by the relationship between the frequency difference of the tones and the corresponding *critical bandwidth*. The maximum perceived dissonance was estimated to occur when the two pure tones were about 1/4 of the critical bandwidth apart, and the intervals were estimated to be consonant, when the frequency difference exceeded the critical bandwidth [24]. Kameoka and Kuriyagawa found that dissonance would increase with the SPL of the stimulus [26], which can be explained by the finding that critical

bandwidth are wider with louder stimuli.

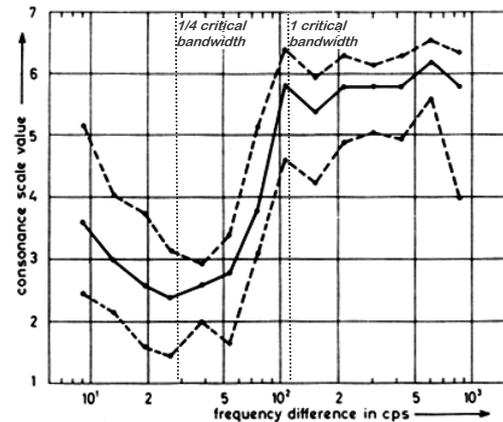


Figure 4: "Consonance rating scores of simple-tone intervals with a mean frequency of 500 Hz as a function of frequency difference between the tones. The solid line corresponds with the median, the dashed curves with the lower and upper quartiles of the scores (11 subjects)." (from Plomp & Levelt [24]). Critical bandwidth annotations added.

The curve in Figure 5 is the consonance measure *SC* computed by the model introduced in the preceding section. The stimuli consist of pure-tone intervals within one octave, with the lowest tone at 440 Hz and the highest at frequencies in the range 440-880 Hz. Each static tone pair was presented in isolation to the model.

The consonance model uses a gammatone filterbank with 64 channels. When a pure tone is swept along the frequency axis, it may lead to fluctuations of the output because the tone is sometimes at the centre of a filterbank channel and sometimes in between two. This may be contributing to the *oscillations of the consonance measure* in Figure 5, which would hence be an artefact of the model. Therefore a *smoothed* version of the curve is also presented in the figure. A filterbank with more overlapping channels has not yet been tested.

In Figure 5, the octave interval (1200 cent) has a higher

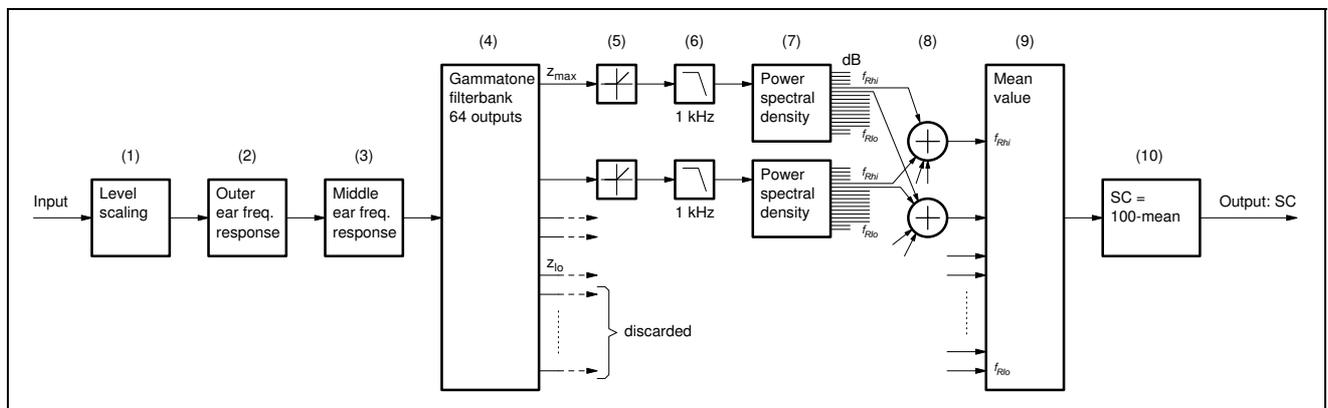


Figure 3: Diagram of the sensory consonance model

modelled consonance value than the unison (0 cent, i.e. in essence only one tone). This may seem counter-intuitive, yet it could be argued that two pure tones, separated by an octave, might easily fuse together and hence be heard as *a single tone with two harmonics*. This phenomenon appears to be confirmed by the psychoacoustic experiments [26] (though not by Figure 4, as we don't know how much the curve would rise when the frequency difference approaches 0 Hz).

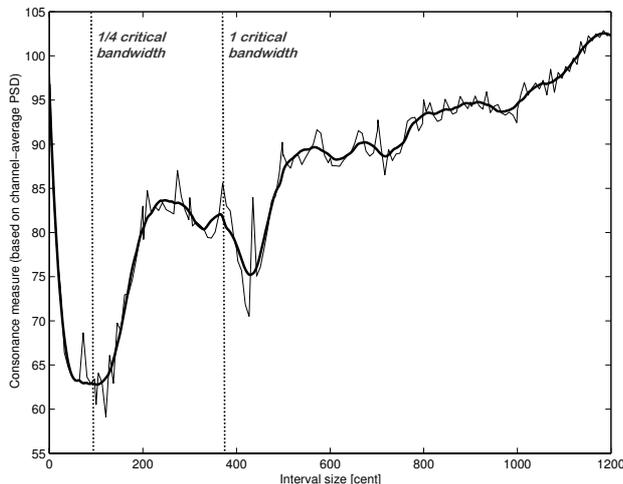


Figure 5: Modelled consonance (*SC*) for intervals of two pure tones. The thin line is the plot of the raw values while the thicker graph represents the data smoothed by a filter. The lower pitch is 440 Hz, the upper is varied from 440 to 880 Hz.

When comparing Figure 4 and Figure 5, please note that the former uses a frequency-difference scale while the latter uses *cent* (frequency ratio). Furthermore, the range 9-900 Hz in Figure 4 corresponds to an interval range of 31-2800 cent, i.e. a little over two octaves, as opposed to only one octave (0-1200 cent) in the modelled consonance curve. Additionally, the 'consonance rating score' is based on a percentage of subjective judgements, whereas the 'consonance measure' is a continuous parameter calculated from a computational auditory model.

With these reservations in mind, a cautious comparison of the two curves in Figure 4 and Figure 5 reveals a *qualitative similarity*. In particular, the consonance *minimum*, or dissonance maximum, for both curves is located at intervals around 1/4 of the critical bandwidth, as indicated on both figures. The critical bandwidth at 500 Hz (~5 Bark) is 115 Hz [15]. In Figure 4, the minimum value, located at ~30 Hz frequency difference between the two pure tones, corresponds to $30/115 = 0.26$ times the critical bandwidth. As the interval size increases towards a whole critical bandwidth, both consonance curves rise. Moreover, as the intervals shrink towards the unison (0 cent), both curves climb smoothly to a local maximum.

In summary, the results of the modelled consonance *SC* of pure-tone intervals are found to be qualitatively similar to consonance/dissonance judgements reported in established psychoacoustic experiments.

In contrast to the pure-tone intervals, the model output for **intervals of tones with a musically pseudo-realistic spectrum shows numerous distinct consonance peaks**. Furthermore, the range of the *SC* measure is doubled, compared to the pure tone intervals. In Figure 6, frequency intervals from unison up to one octave are sampled, including those of the equal tempered scale (corresponding to integer multiples of 100 cent). Furthermore, certain pure intervals are added. The intervals consist of the type of harmonic tones described in section 2.1.

One striking feature of the consonance curve in Figure 6 is how dissonant most of the tempered intervals are, with typical values $SC < 30$, compared to the 'pure' intervals. For instance, the *pure* perfect fifth (702 cent) has a $SC = 87$, whereas the *equal tempered* perfect fifth (700 cent) is at $SC = 61$. The tempered perfect fifth is still quite consonant though, compared to other intervals.

The only difference in the two groups of stimuli, underlying Figure 5 and Figure 6, is the presence of the harmonic partials in the tones of the latter. Yet the consonance curves are quite different in both shape and range. Therefore, according to the model **the timbre of the tones has a significant effect on the sensory consonance of tone intervals**.

To further demonstrate the results of the consonance model developed in this work, **a digital video was created**. The video consists of the modelled consonance curve of Figure 6, *annotated* such that each sampled interval is in turn highlighted by a red marker. This animation was merged with a soundtrack composed of one second of synthesized audio for each sampled interval, consisting of harmonic tones – the same sound that was used as input for the model. The modelled consonance can thus be *seen* while simultaneously the stimulus is *heard*.

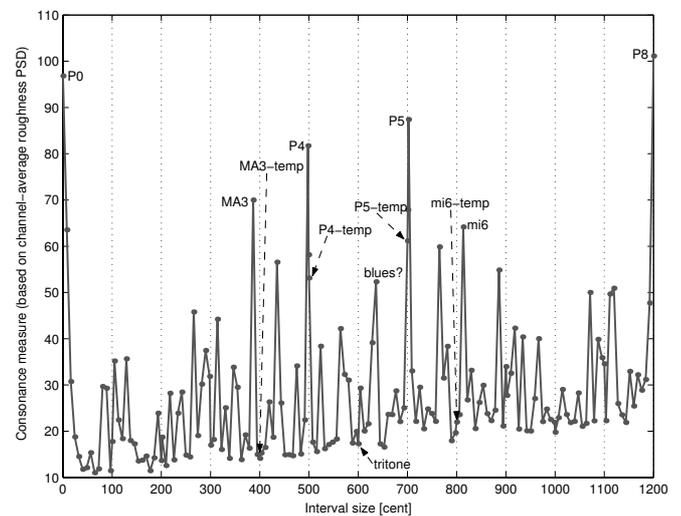


Figure 6: Modelled consonance for intervals of two harmonic tones. Each round marker on the graph indicates a specific interval whose *SC* value has been computed. The pitch range is as in Figure 5. Vertical dotted lines show the equal tempered scale intervals. The marked tempered intervals have names containing 'temp', the other names correspond to pure intervals.

5. CONCLUSIONS

A measure of sensory consonance was developed, employing a scheme similar to an established auditory model of pitch, and motivated by knowledge of critical bandwidth and the roughness sensation. The behaviour of the consonance measure for musical intervals was in agreement with conventional musical knowledge, even though the underlying model was based on psychoacoustics and devoid of music-specific presumptions. Thereby the consonance measure could be employed as a perceptually related feature (extractor) for musical signals.

The consonance measure for the equal tempered intervals, which are generally used in western music, was significantly lower – i.e. more dissonant – than for the corresponding pure intervals; certain tempered intervals were no more consonant than some of the out-of-scale intervals. This supports the supposition that the perceived musical consonance of tone intervals, especially in a musical context, depends on other significant factors than the sensory consonance modelled here.

Only static, periodic and isolated stimuli have been considered here. For application in a musical context, the consonance model would need to handle dynamic signals. By implementing a sliding-window analysis as an extension to the model, also time-varying signals could be used as stimuli.

The consonance measure computed by the model was shown to depend both on the frequency ratio between the tones, and on harmonic spectrum (corresponding to the timbre) of the tones. The former result is in agreement with the long-established theory of consonance, and the latter must have been realised centuries ago at least by musicians and musical orchestrators; yet in traditional music notation and analysis, is harmony not – even today – treated independently of the timbral aspects of music ?

6. REFERENCES

- [1] Skovenborg, E., "A Pitch Model of Consonance", chapter 2 in M.D.Vestergaard (ed.) *Collection Volume, Papers from the 1st Seminar on Auditory Models*, Ørsted-DTU, Dept. of Acoustic Technology, publication no. 55. ISSN 1395-5985, 2001.
- [2] MPEG-7, "Information Technology – Multimedia Content Description Interface – Part 4: Audio (part of MPEG-7)", ISO/IEC Committee Draft 15938-4. ISO/IEC JTC1/SC29/WG11 (MPEG), 2001.
- [3] Lindsay, A.T. & Herre, J., "MPEG-7 and MPEG-7 Audio – An Overview", *J. Audio Eng. Soc.* (49), 589-594, 2001.
- [4] Todoroff, T., "Control Based on Sound Features" ch. 12 in Zölzer, U. (ed.) *DAFX – Digital Audio Effects*, John Wiley & Sons, Chichester, 2002.
- [5] Verfaillie, V & Arfib, D., "A-DAFX: Adaptive Digital Sound Effects", *Proc. of DAFX-01*, Limerick, 10-14, 2001.
- [6] Nielsen, S.H., "Realtime Control of Audio Effects", *Proc. of DAFX-99*, Trondheim, 1999.
- [7] Hartmann, W.M., "Pitch, periodicity, and auditory organization", *J. Acoust. Soc. Am.* 100(6), 3491-3502, 1996.
- [8] Lyon, R. & Shamma, S., "Auditory Representations of Timbre and Pitch", ch. 6 in Hawkins, H.L. et al. (eds.) *Auditory Computation*. Springer Handbook of Auditory Research, vol. 6. Springer, New York, 1996.
- [9] Slaney, M. & Lyon, R.F., "A perceptual pitch detector", *Proc. 1990 Int. Conf. on Acoustics, Speech, and Signal Processing* (pp. 357-360), Albuquerque: IEEE, 1990.
- [10] Meddis, R. & Hewitt, M.J., "Virtual pitch and phase sensitivity of a computer model of the auditory periphery. I: Pitch identification", *J. Acoust. Soc. Am.* 89(6), 2866-2882, 1991.
- [11] Meddis, R. & O'Mard, L., "A unitary model of pitch perception", *J. Acoust. Soc. Am.* 102(3), pp.1811-1820, 1997.
- [12] Härmä, A., "HUTear Matlab Toolbox, version 2.0", Internet site, URL: <<http://www.acoustics.hut.fi/software/HUTear/>>, 2000.
- [13] Hawkins, H.L., McMullen, T.A., Popper, A.N. & Fay, R.R. (eds.), *Auditory Computation*, Springer Handbook of Auditory Research, vol. 6. Springer, New York, 1996.
- [14] Houtsma, A.J.M., Rossing, T.D. & Wagenaars, W.M., "Auditory Demonstrations (audio CD)", Prepared at the Institute for Perception Research (IPO), Eindhoven, The Netherlands; supported by the Acoust. Soc. Am. (ASA). Philips CD 1126-061, 1987.
- [15] Zwicker, E. & Fastl, H., *Psychoacoustics: Facts and Models*. (2.ed.), Springer Series in Information Sciences 22. Springer-Verlag, Berlin, 1999.
- [16] Terhardt, E., "Pitch, consonance, and harmony", *J. Acoust. Soc. Am.* 55(5), 1061-1069, 1974.
- [17] Terhardt, E., "The concept of musical consonance: A link between music and psychoacoustics", *Music Perception*, 1, pp.276-295, 1984.
- [18] Blackwood, E., *The Structure of Recognizable Diatonic Tunings*, Princeton University Press, 1985.
- [19] Rossing, T.D., *The Science of Sound*. (2.ed.), Addison Wesley, New York, 1990.
- [20] Roederer, J.G., *The Physics and Psychophysics of Music – an Introduction*. (3.ed.), Springer-Verlag, Berlin, 1995.
- [21] Sadie, S. (ed.), *The Grove Concise Dictionary of Music*, Macmillan, Bath, UK, 1994.
- [22] Aures, W., "Der sensorische Wohlklang als Funktion psychoakustischer Empfindungsgrößen", *Acustica* (58), 282-290, 1985.
- [23] Aures, W., "Berechnungsverfahren für den sensorischen Wohlklang beliebiger Schallsignale", *Acustica* (59), 130-141, 1985.
- [24] Plomp, R. & Levelt, W.J.M., "Tonal consonance and critical bandwidth", *J. Acoust. Soc. Am.* 38(2), 548-560, 1965.
- [25] Plomp, R., *Aspects of Tone Sensation*, Academic Press, London, 1976.
- [26] Kameoka, A. & Kuriyagawa, M., "Consonance Theory I: Consonance of Dyads", *J. Acoust. Soc. Am.* 45: 1451-1459, 1969.